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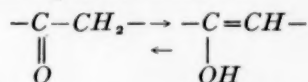
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THE ORIGIN OF COLOR. I

BY E. C. C. BALY AND A. W. STEWART¹

In the previous paper it was shown how the presence of two carbonyl groups in juxtaposition gives rise to the appearance of an absorption band in the spectrum very much nearer to the red than that which is produced by the process known as² enol-keto tautomerism,² or a reversible equilibrium such as



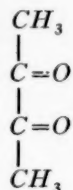
In the case of pyruvic ester $CH_3CO COOEt$, in which it was first discovered, the absorption band is situated at a frequency of 3100, while the band due to enol-keto tautomerism is always very near to 3800. Now, in pyruvic ester there is only one true carbonyl group, for it is well known that the $-CO-$ group of a carboxyl radicle is not endowed with all the properties usually appertaining to this group. In order to investigate this process more fully, we have examined the absorption spectra of a series of compounds which contain two true carbonyl groups in juxtaposition, and we then found that the new absorption band is still nearer to the red than in the case of pyruvic

ester. Camphorquinone C_8H_{14} $\begin{array}{c} \diagup C=O \\ | \\ \diagdown C=O \end{array}$ was dealt with in the pre-

¹ Carnegie Research Fellow.

² Baly and Desch, *Astrophysical Journal*, 23, 110, 1906.

vious paper where its absorption curve was given; a very persistent absorption band is exhibited with its head at a frequency of 2100. The simplest compound of this type is of course diacetyl



and the absorption curve of this substance is shown in Fig. 1, where the new absorption band appears at a frequency of 2400. Since these two compounds, camphorquinone and diacetyl, show absorption bands in the visible blue ($\lambda = 4760$ and 4170 t.-m. respectively), they are naturally colored yellow, and clearly therefore the process which produces these absorption bands is the origin of color in the case of these compounds.

In the previous paper we showed that the origin of the new absorption band is to be found in some form of oscillation between the residual affinities of the oxygen atoms of the carbonyl groups, and for this oscillation we proposed the name *isorropesis*. Before dealing with the theoretical aspect, we may say that we have extended our observations to include many other compounds containing two carbonyl groups in juxtaposition, and in every case we find the new absorption band present. For example, the absorption

curves of acenaphthenequinone $C_{10}H_8 \begin{array}{c} \diagup \text{C}=\text{O} \\ | \\ \diagdown \text{C}=\text{O} \end{array}$ and phenanthraqui-

none $C_{12}H_8 \begin{array}{c} \diagup \text{C}=\text{O} \\ | \\ \diagdown \text{C}=\text{O} \end{array}$ are shown in Fig. 2; isatin, whose spectrum

has already been recorded by Hartley and Dobbie,¹ is another case in point and shows the same absorption band with head at a fre-

quency of 2400. The formula of isatin $C_8H_4 \begin{array}{c} \diagup \text{CO} \\ \diagdown \text{NH} \end{array} \text{CO}$ at once explains the appearance of the band, as there are present two carbonyl groups in juxtaposition.

¹ Chem. Soc. Trans., 75, 640, 1899.

Another very interesting case of an α -diketone is that of benzil, $C_6H_5.CO.CO.C_6H_5$, the absorption-curve of which is shown in Fig. 3. It is well known that the oscillating double linking of the

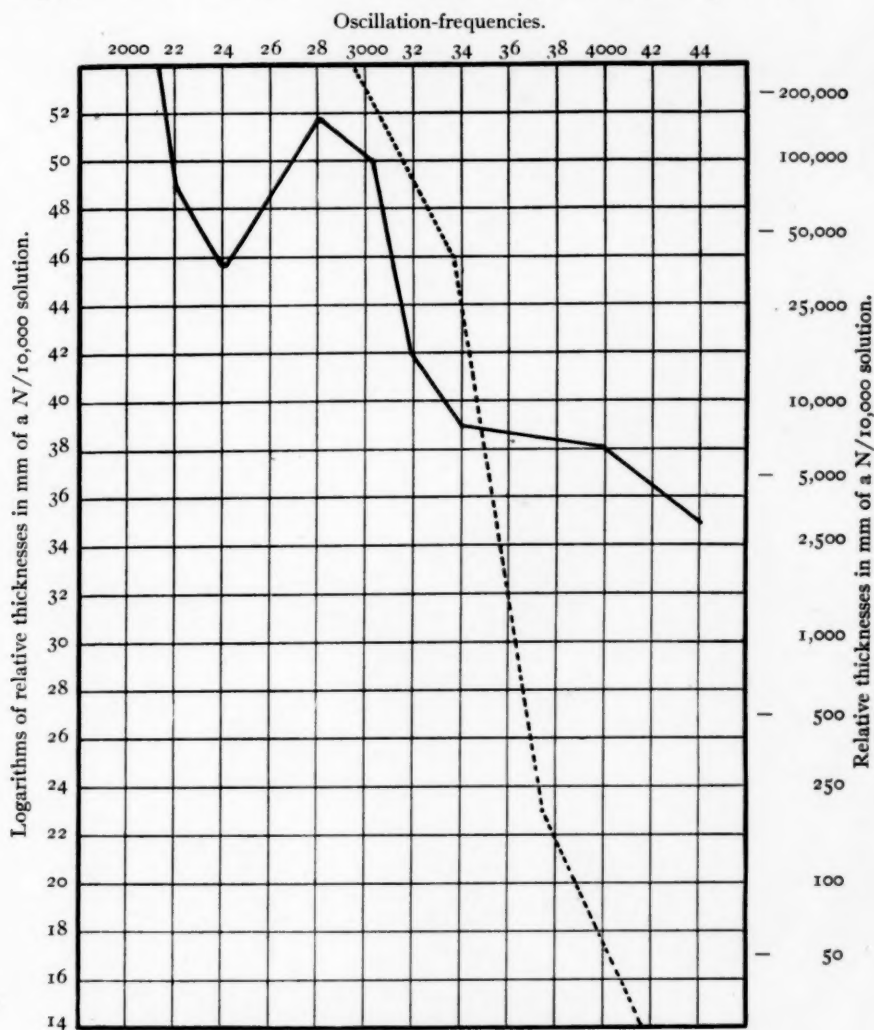


FIG. 1.—Diacetyl (full curve): Diacetyl dioxime (dotted curve).

benzene ring, or the benzenoid tautomerism, produces absorption bands which have about the same frequency as the absorption band due to enol-keto tautomerism. In the case of benzene itself there

are seven narrow absorption bands¹ with heads at frequencies of 3725, 3765, 3830, 3915, 4025, 4110, and 4200. These absorption

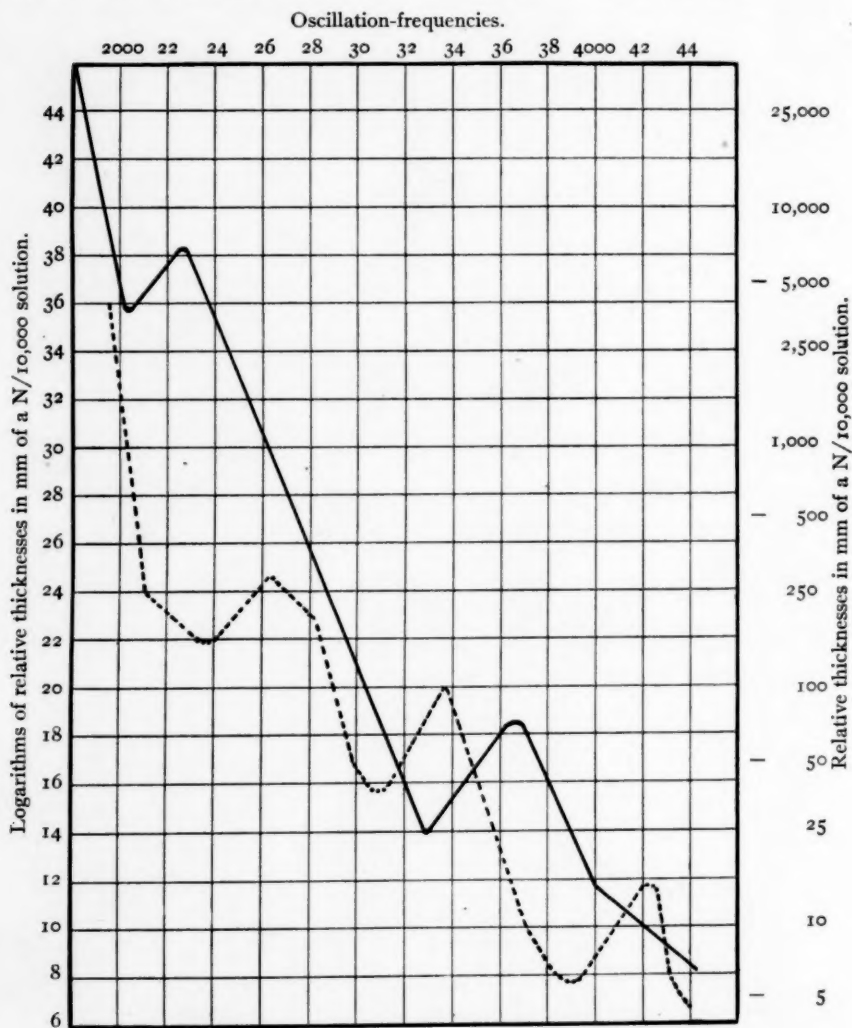


FIG. 2.—Acenaphthenequinone (full curve).
Phenanthraquinone (dotted curve).

bands are considerably modified by substituting different groups for the hydrogen atoms, especially if the substituent groups possess

¹ Baly and Collie, *Ibid.*, 87, 1332, 1905.

residual affinity. Thus acetophenone, $C_6H_5.CO.CH_3$, in which one hydrogen of benzene has been replaced by the acetyl group $-CO.CH_3$, shows an absorption very different from that of benzene.

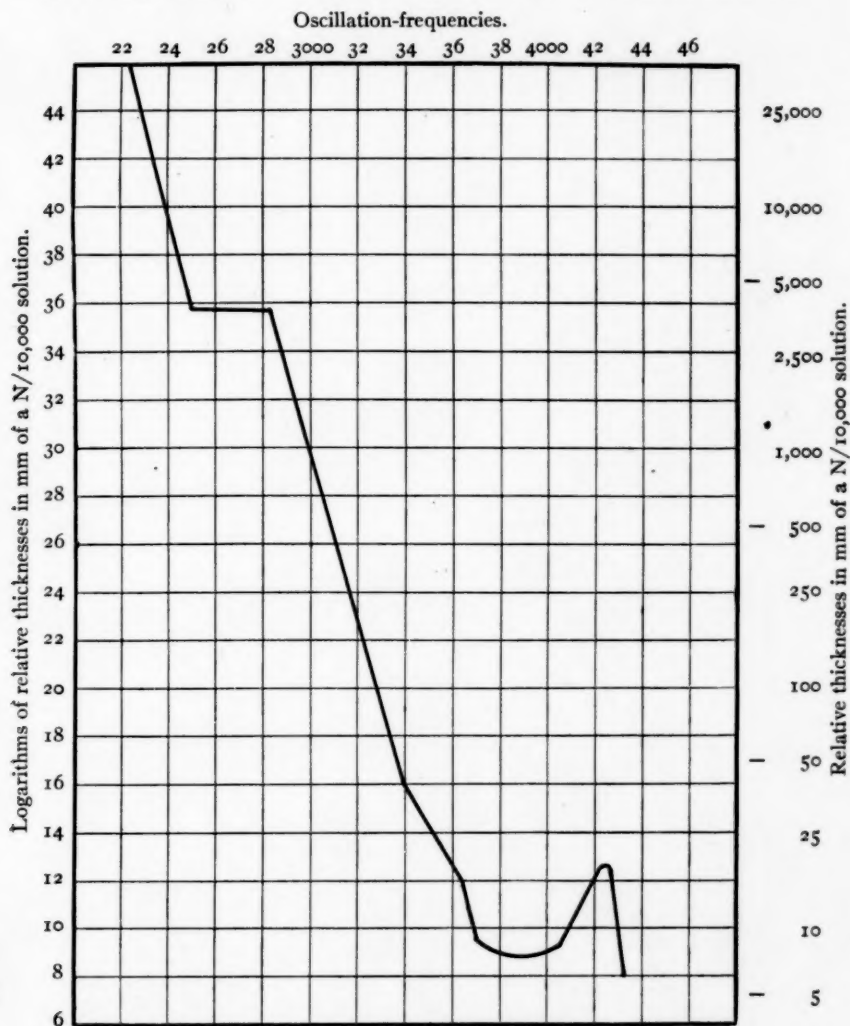


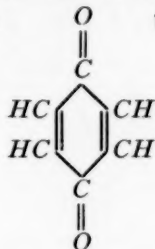
FIG. 3.—Benzil.

All the characteristic absorption due to the benzene ring has disappeared, owing no doubt to the fact that the residual affinity of the benzene ring has been fixed by the attraction between the car-

bonyl group and the atoms of the ring. This accounts for the fact that the carbonyl group of acetophenone is unusually inactive toward sodium bisulphite, etc., because the group does not readily pass into the nascent state necessary to the formation of additive compounds. It might be expected, therefore, that in benzil the residual affinities of the two carbonyl groups would each be occupied with and fixed by the adjacent benzene nucleus, and that therefore no isorropesis would occur. In reference, however, to the absorption-curve of this substance (Fig. 3) it will be seen that in the region of least concentration there is an absorption band with head at a frequency of 3900. The presence of this band argues that the benzenoid tautomerism is undoubtedly present to a small extent. For this reason we may conclude that the residual affinities of the carbonyl groups are not entirely fixed, and that a small amount of isorropesis between them is possible. It is evident that this conclusion is justified from an inspection of the upper portion of the absorption-curve of benzil, where a shallow band with head at a frequency of 2650 appears. The existence of this band shows that isorropesis is taking place, and its shallowness proves that it is present to a small extent only. It may be noticed that the yellow color of benzil is not very pronounced, and readily disappears in dilution. The measurements of the additive capacity of the benzil carbonyl groups made by Petrenko-Kritschenko agree very closely with the above observation.

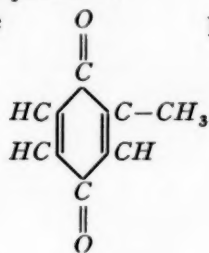
There is thus little doubt that the color of the compounds, diacetyl, camphorquinone, acenaphthenequinone, phenanthraquinone, isatin, and benzil is due to the two carbonyl groups in juxtaposition, since this configuration gives rise to a new type of oscillation or isorropesis between the residual affinities upon the two adjacent oxygen atoms. The most striking application of this principle is in the case of the true benzenoid quinones, for in these compounds, which are all strongly colored, we have a type of compound resembling an α -diketone, and in these compounds, too, the new absorption band is exhibited showing the undoubted existence of the process of isorropesis between the quinonoid oxygen atoms. Quinone itself was dealt with in the preceding paper and its absorption-curve there reproduced. It may, however, be again emphasized that all the chemical

evidence supports the view that the two para positions in the benzene ring are very close together, and that we should therefore expect the two =CO groups of quinone to have the same prop-

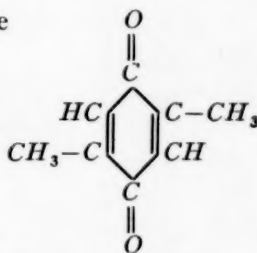


erties as those of an α -diketone. We have also examined the absorption spectra of

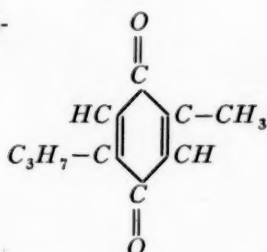
toluquinone



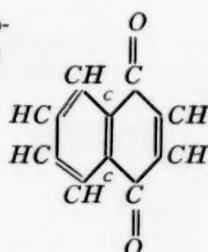
para xyloquinone



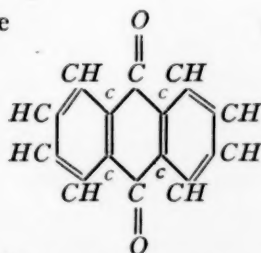
thymoquinone



α naphthoquinone




and anthraquinone



and find that the same

absorption band is present in each case. The absorption-curves of the two first compounds are reproduced in Figs. 4 and 5; the remaining compounds show very similar curves. There is no

doubt therefore that the process of isorropesis exists in the quinones and is the origin of color of these compounds.

Now, Armstrong has developed a theory of color in which he attributes this property to the quinonoid linking;  the impor-

tance of these results in relation to his theory is manifest. They would seem to supply the key to his generalizations and at the same time to explain the colors of many substances which are difficult of interpretation by Armstrong's theory alone. Armstrong's theory gives no explanation of why color is produced by the quinonoid linking; there is no esoteric value in any of the linkings of the formula as light-absorbing centers. The results we have given, however, show that when the quinonoid configuration exists, isorropesis is set up between the residual affinities of the groups in the para position, with the result that an absorption band is developed in the visible region of the spectrum, producing a yellow or orange color.

In considering the whole question of color, there is no doubt that the new principle may be extended to include every case; that is to say, that isorropesis may occur between any adjacent atoms possessing residual affinity. It must be remembered, however, that in order for the new oscillation to take place, it is absolutely necessary for some exciting or disturbing influence to be present. For example, in the group $\begin{array}{c} -C-C- \\ || \quad || \\ O \quad O \end{array}$ of the α -diketones, each oxygen

atom possesses a definite amount of residual affinity, and it is evident that no oscillation can arise between these atoms unless one or both residual affinities are disturbed. Now, in diacetyl, $CH_3-CO-CO-CH_3$, this influence is furnished by the hydrogen atoms of the methyl groups. There is an attraction between the hydrogen and oxygen atoms, with the result that the residual affinities on the latter tend to be altered. We have direct evidence of this potential enol-keto tautomerism in the absorption-curve of diacetyl (Fig. 1), for the curve shows a sudden extension at the ordinate 38. This extension undoubtedly means the incipient formation of an absorption

band which occupies the position of the band due to the tautomerism of a labile hydrogen atom.¹ Clearly therefore the residual affinities of the two oxygen atoms are being slightly disturbed, and it is owing

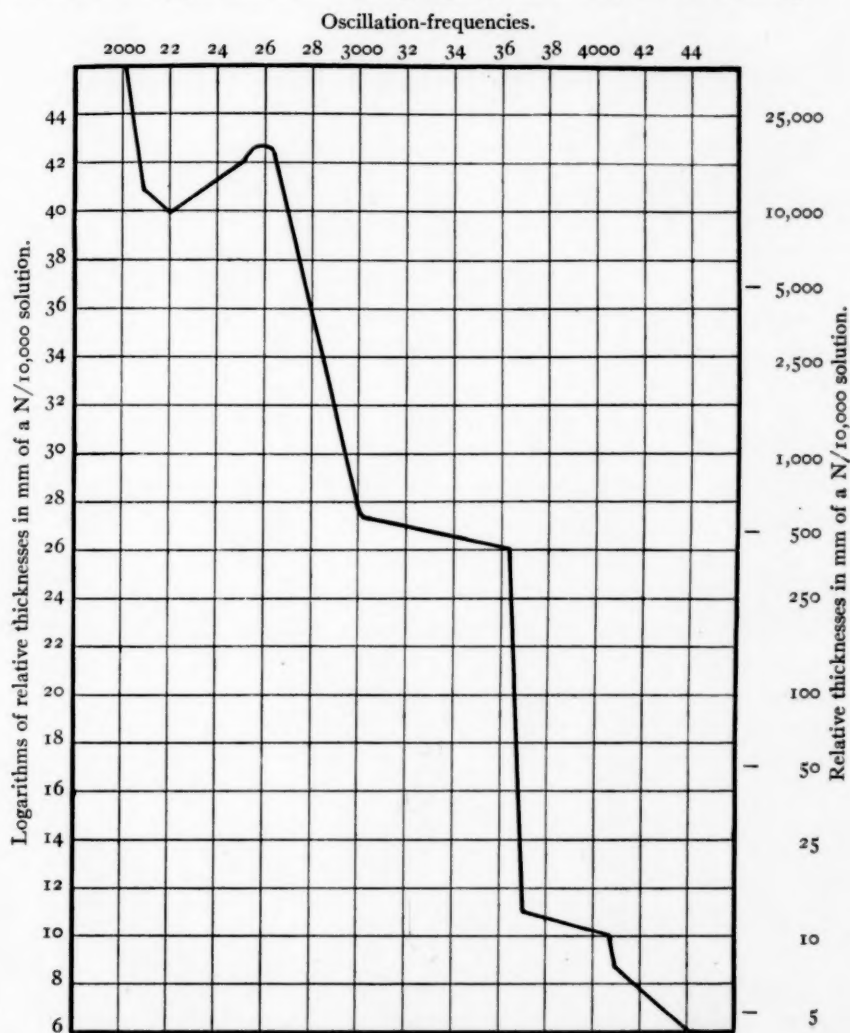
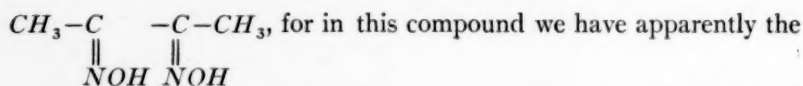


FIG. 4.—Toluquinone.

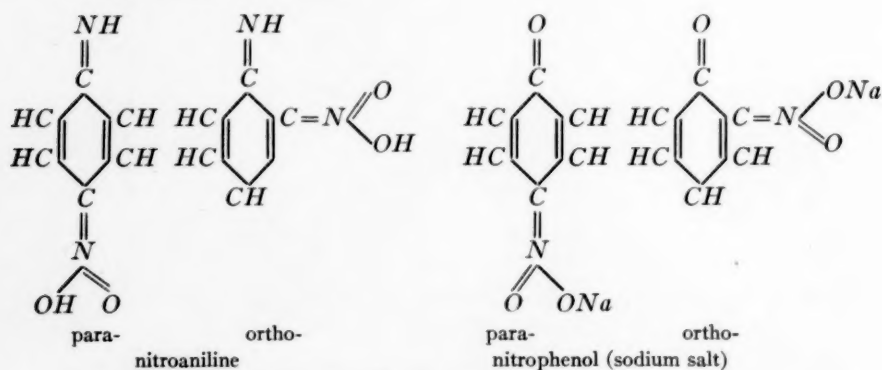
to this disturbance that the new oscillation or isorropesis takes place. We may now understand why diacetyl-dioxime is colorless

¹ Baly and Desch, *loc. cit.*



condition for color, and yet only general absorption is exhibited. The residual affinity of the nitrogen atoms exerts no attraction on the hydrogen atoms of the methyl groups, and therefore is not disturbed in any way. No isorropesis therefore is set up and the compound is colorless. The absorption-curve of diacetyl-dioxime is shown in Fig. 1. It follows from this that all assumptions that two compounds must have essential differences in constitution if one is colored and the other white are untrustworthy.

It is evident that, if our generalizations upon color are correct—namely, that isorropesis occurs between the residual affinities of unsaturated atoms in juxtaposition, there is a large field for investigation among compounds of the quinonoid type in which the oxygen atoms of quinone are replaced by other unsaturated atoms. It should be noticed that in the quinones the necessary disturbing influence is provided by the tautomerism of the benzene ring, and that we are not entirely dependent upon the near presence of hydrogen atoms.¹ We have investigated the nitroanilines and the nitrophenols, in which unsaturated nitrogen atoms are present.² The color of the former and of the latter in alkaline solution has been accounted for by Armstrong on the assumption that they exist in the quinonoid form thus



¹ Stewart and Baly, *Chem. Soc. Trans.*, **89**, 618, 1906.

² Baly, Edwards and Stewart, *Ibid.*, **89**, 514, 1906.

Considerable difficulty was met with by Armstrong in the case of the meta compounds, because it is impossible to imagine the static

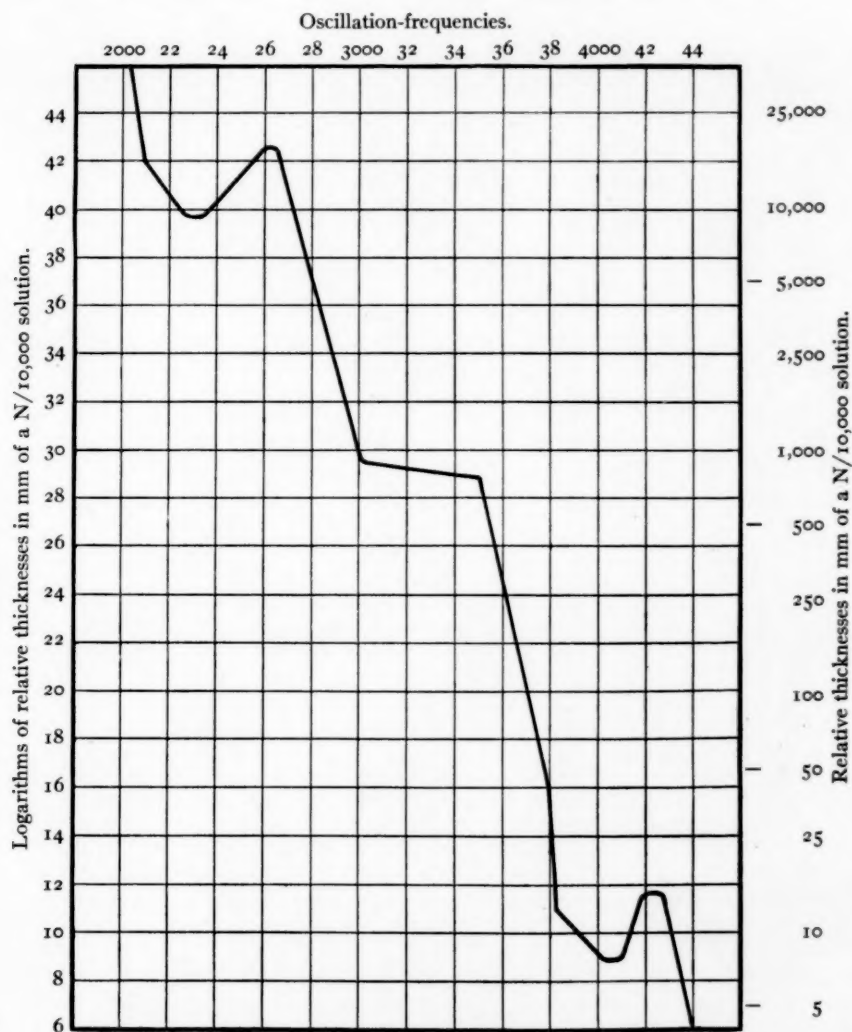
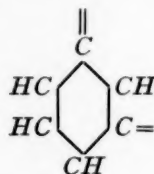
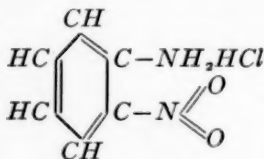


FIG. 5.—Para-xyloquinone.

existence of a linking of this type, as can readily be seen from the formula, there being no satisfactory way of linking the four remaining carbon atoms of the ring,



In Fig. 6 are reproduced the curves of meta- and paranitroaniline, and they show the presence of the absorption band due to isorropesis; this band has, however, much less persistence in the case of the meta compound. The absorption of the ortho compound is practically identical with that of the para derivative. Now, these substances all give colorless solutions in the presence of strong hydrochloric acid; the absorption of these solutions is the same in each case and is exemplified by the curve in Fig. 7. There is no doubt from this that in the presence of hydrochloric acid the compounds possess the structure of the true hydrochloride, e. g.:



and that in neutral solution they possess the quinonoid form as given above. Isorropesis therefore occurs in these compounds between two unsaturated nitrogen atoms. Similarly, it has been proved that isorropesis occurs between the unsaturated nitrogen and oxygen atoms in the quinonoid forms of the nitrophenols. In these latter compounds, as in the case of the nitroanilines, the isorropesis is very much less in the meta than in the ortho and para isomers, showing that the metaquinonoid form undoubtedly exists, but only to a small extent. It is evident, therefore, that some restraining influence is at work against the formation of the meta-quinone. It was stated above that the static existence of a meta-quinone is impossible, but we have in these compounds undoubted evidence of the meta-quinone existing in proportionately small amounts; we may conclude, therefore, that this form has not a static but only a transitory existence. This may be explained as follows.

The space-formula proposed by Collie¹ has the advantage of representing the benzene molecule as a system of atoms in a state

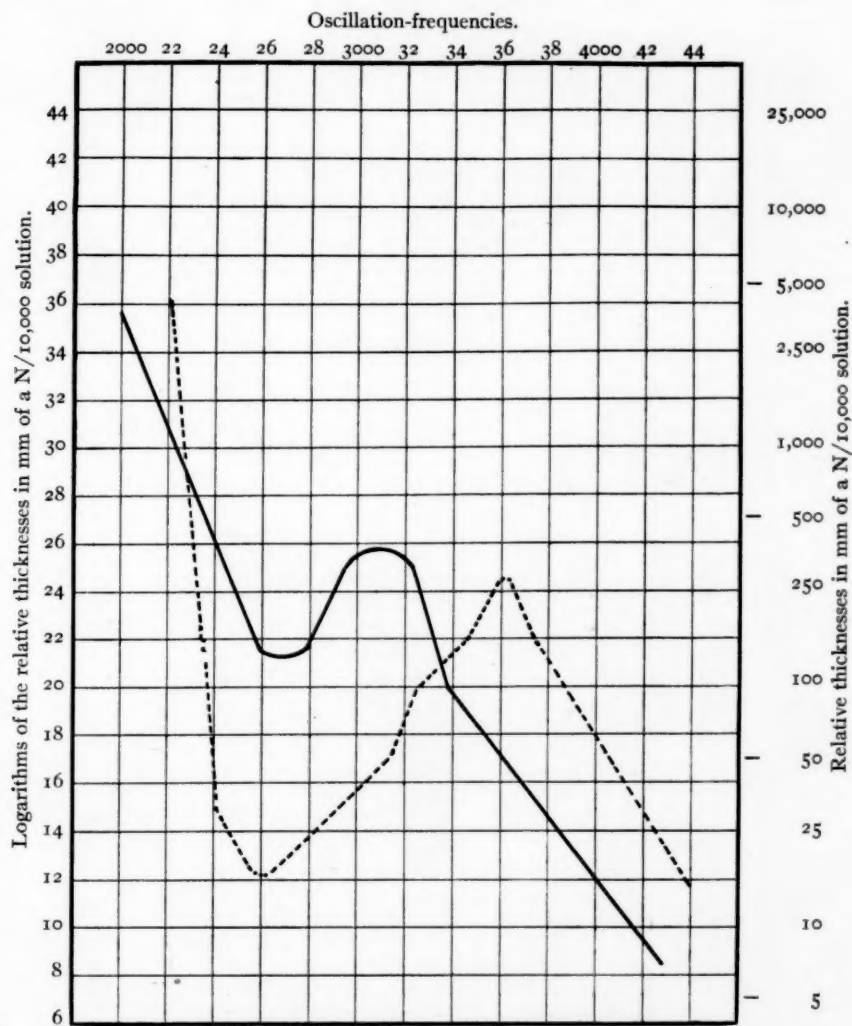


FIG. 6.—Meta-nitroaniline (full curve); Para-nitroaniline (dotted curve).

of continual vibration, and by this means it was possible to express all the various formulæ which had then been put forward as phases

¹ *Chem. Soc. Trans.*, 71, 1013, 1897.

of one formula. We consider that this idea of a system in motion is extremely important, but at the same time it is evident that vibra-

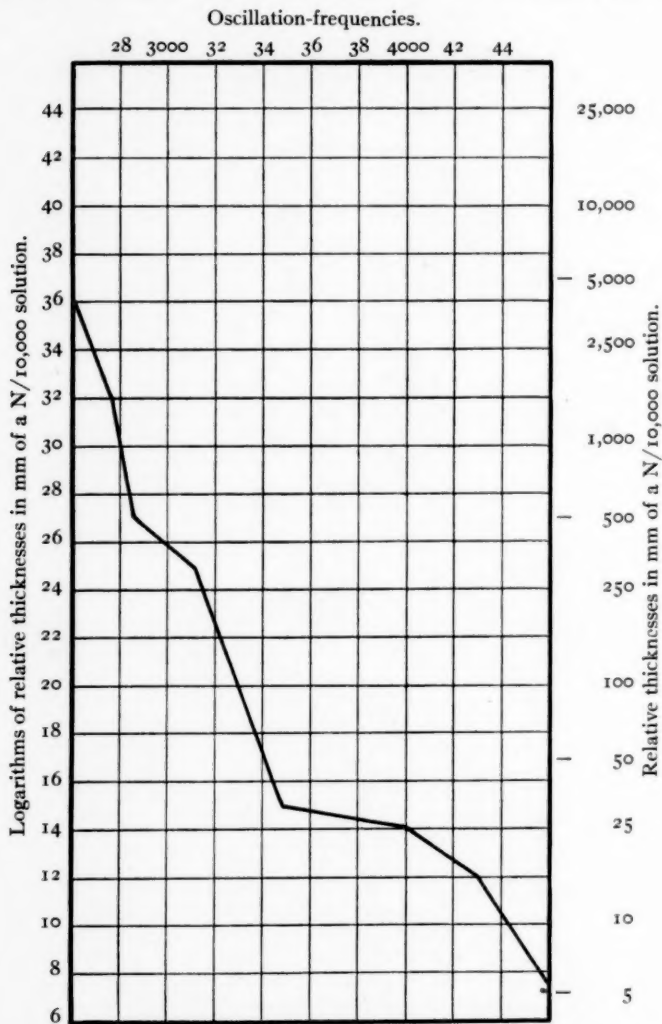
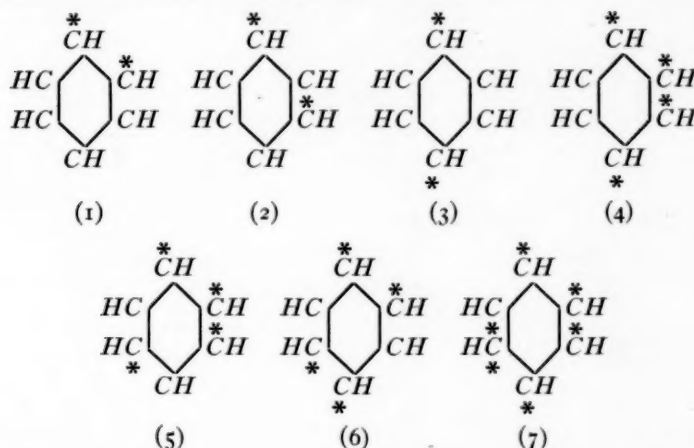


FIG. 7.—Ortho-nitroaniline in hydrochloric acid.

tions of the atoms not expressly described in Collie's original paper must be introduced in order to bring the theory into line with the spectroscopic and chemical evidence now at our disposal. It has been shown¹ that benzene gives seven very similar and closely situ-

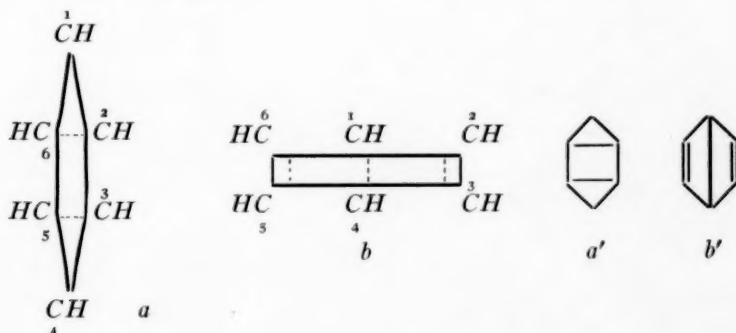
¹ Baly and Collie, *loc. cit.*

ated absorption bands, and it was pointed out that the formation of these can be accounted for by assuming that each band is due to a separate make-and-break of linking between the carbon atoms of the ring. There are seven such makes and breaks possible, as can be seen from the following figures, the asterisks being attached to those atoms which are changing their linking:



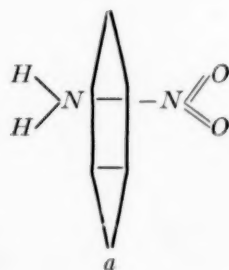
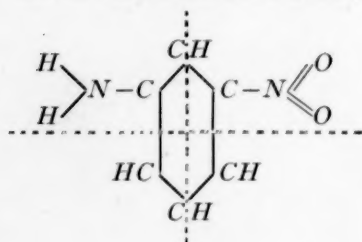
It will be seen that in case (2) a single meta-linking is being formed or broken; this throws some light on the possibility of the existence of meta-quinones.

Now, in order to bring the seven phases into existence, it is necessary to assume the displacement of the carbon atoms of the ring, and we can do this in the simplest way possible—that is to say, by the ordinary vibration as is accepted by any elastic ring. Thus we may say that the benzene ring is pulsating between the two displaced forms *a* and *b*

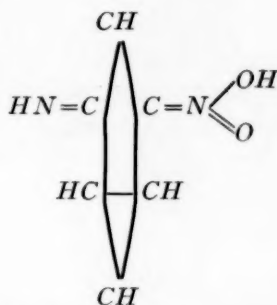


Each carbon atom possesses residual affinity, and consequently in the condition represented in *a*, when the atoms 2 and 6 and the atoms 3 and 5 are brought close together, these residual affinities will produce linkings as shown by the dotted lines. The atoms 1 and 4, however, are far removed from one another and from the other atoms, and are therefore unsaturated. On the other hand, when the ring has passed into the other phase *b*, then the three atoms 2, 1, and 6 come very close to the three atoms 3, 4, and 5 respectively, and linking may be considered to be formed between these pairs of atoms. The linkings existing in phases *a* and *b* are shown for greater convenience on the ordinary hexagons in *a'* and *b'*. As the ring is pulsating between the forms *a* and *b*, many of the seven phases of linking-change described above will be obtained. For example, let us consider the ring to have reached the form *b*; as it starts opening, the first break will occur between the atoms 1 and 4, giving phase No. 3. This will be followed by the breaking of the two ortho-linkings 2:3 and 5:6, giving phases Nos. 3 and 6. When the ring passes through the half-way stage—that is, the circular form—then we shall have the centric formula, with the result that phase No. 7 is produced. We can in this way account for phases 1, 2, 3, 6, and 7; Nos. 4 and 5 can readily be understood if the motions described above are slightly interfered with by collisions between adjacent molecules. In the above it was assumed that the displacement takes place so that atoms 1 and 4 are at the ends of the ellipse in the form *a*, but in general the displacement will take place along any of the three possible axes.

This scheme of displacement of the benzene ring renders it perfectly possible for meta-quinones to have a transitory existence. Let us take meta-nitroaniline,



and let the displacement take place along the dotted axes, when we shall obtain phase *a*. When in the form *a*, then the metaquinone can exist thus:



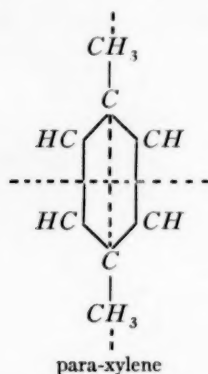
It must be remembered that this meta-quinone can exist only when the displacement occurs in the way shown. It is not therefore necessary to conceive the static existence of a metaquinone, but it is clearly possible for such a linking to exist during part of the motions of the ring.

It has been stated above that meta-nitroaniline and that meta-nitrophenol in alkaline solution exist only partly in the quinonoid form. Inasmuch as a special form of vibration is necessary in order that the meta-quinone may exist, we may say that in this fact is to be found the undoubted restraining influence against the formation of the meta-quinone referred to above.

This pulsation of the benzene ring explains very satisfactorily many of the characteristic physical and chemical properties of benzene and its derivatives. The explanation of the chemical properties need not be detailed here,¹ but one most striking result observed in the absorption spectra of disubstituted benzene derivatives² is readily accounted for. In these compounds the para isomer is always more symmetrical than the ortho and meta isomers; that is to say, the internal motions of the benzene ring are less disturbed by the para- than by either the ortho- or meta-substitution. This fact is clearly explained by the theory of the pulsating ring, because it is evident that in a compound such as para-xylene the vibration will take place very readily along the dotted axes shown in the figure

¹ Cf. Baly, Edwards, and Stewart, *loc. cit.*

² Baly and Ewbank, *Chem. Soc. Trans.*, **87**, 1355, 1905.

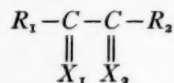


In the ortho- and meta-compounds the unsymmetrical loading of the ring will to a great extent militate against the vibration of the ring, with the result that the ring is distorted and the several absorption bands become confused.

Again, this theory of pulsation readily explains the well-known fact that the two para positions are very close together.

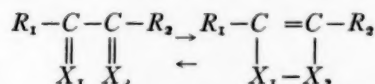
A strong point in favor of this hypothesis is its simplicity. The motion described is the simplest possible, and is the form of vibration adopted by any elastic ring—as, for example, a bell when struck.

These results leave no doubt that when two carbonyl groups are adjacent to one another in a molecule, a new free period of vibration is established; and, further, that when both the groups are true carbonyl as distinct from carboxyl carbonyl groups, the frequency of the new free period is situated in the visible region so that the substance is colored. In general, our results go to prove that the new free period or isorropesis is caused by the existence of the linking:

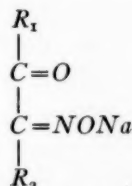


with the proviso that the residual affinity as expressed by the $\begin{array}{c} -C- \\ \parallel \\ X \end{array}$

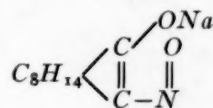
group is disturbed by the influence of the groups R_1 and R_2 . We have attempted to express the process of isorropesis chemically by stating that the new free period is connected with the equilibrium expressed by



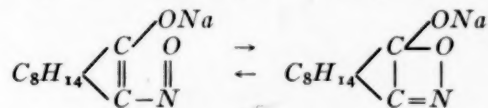
Quite recently¹ we have obtained some experimental evidence in favor of the static existence of the ring form *b*. It is well known that the substances known as the *iso*-nitroso compounds are yellow in alkaline solution. This color, as we have shown, in conjunction with Miss Marsden, is due to the isorropesis occurring with the form



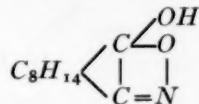
the necessary disturbing influence being provided by the hydrogen atoms upon the radicle R_1 , as regards the $-CO-$ group and by the fact that the sodium atom is labile, in the case of the $\begin{array}{c} | \\ C=N \\ | \end{array}$ grouping. Now, in the case of *iso*-nitroso-camphor, the stable form of this compound is yellow in alkaline solution owing to the isorropesis occurring with the form



This may be expressed chemically by the equilibrium



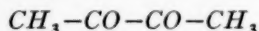
There is direct chemical and spectroscopic evidence that stable *iso*-nitrosocamphor in neutral solution has the form



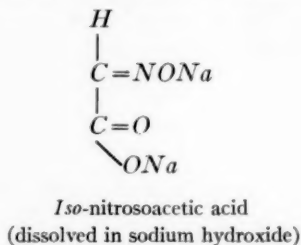
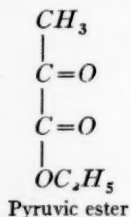
¹ Baly, Marsden, and Stewart, *Chem. Soc. Trans.*, **89**, 1906.

so that our original method of chemically expressing isorropesis has found experimental verification in the case of *iso*-nitroso-camphor.

At the present time we have no physical explanation to offer of the new free period; there is, however, another way of looking at the phenomenon which perhaps may throw more light upon the process. Taking the simplest case of diacetyl

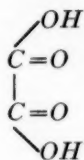


there are two CH_3-CO- groups in juxtaposition, and each one of these, by virtue of the potential enol-keto tautomerism they possess, causes or tends to cause the appearance of an absorption band in the ultra-violet. Inasmuch as these two groups mutually influence one another, it is possible that the two free periods in the ultra-violet may so far interfere or combine together to give a new free period in the visible region. On these grounds, therefore, we should look upon the acetyl group or any other group showing enol-keto tautomerism and the benzene nucleus showing benzenoid tautomerism as being potential color systems. The juxtaposition of two of these systems in certain definite ways gives rise to isorropesis or the combination of the two systems to give a new free period. In the compounds described above the new free period is situated in the visible region, so that the substances have visible color. It must be remembered that the conditions may occur in which the isorropic free period is not in the visible region; in this case the substance would not be colored. Such a condition occurs in both pyruvic ester and in *iso*-nitrosoacetic acid in alkaline solution.



In the case of both these compounds the frequency of the isorropic free period is about 3100, which is not in the visible region, and the substances are colorless. It appears that the presence of the hydroxyl oxygen next to the carbonyl group produces this effect, and experi-

ments are at present in progress with a view of explaining this influence. It is interesting to note that the frequency of 3100 obtained with the above two compounds is exactly half-way between the frequency of the isorropesis of diacetyl (2400) and of the enol-keto tautomerism absorption (3800), and further that no isorropesis occurs in oxalic acid where both the carbonyl groups form part of a carboxyl group:



Oxalic acid

It is very noteworthy that the wave-length of the free period of vibration established by isorropesis is about the same as that emitted by the simpler fluorescent organic substances ($\lambda=4800-\lambda=4000$). It may be that there is an intimate connection between fluorescence and isorropesis, and that the former is only a manifestation of the latter. There is nothing inherently improbable in this idea. In both cases, visible color and fluorescence, the free period is established by the isorropesis; in the former case the free period is established by the isorropesis and excited by the incident light and we have absorption, while in the latter case the free period is established and excited by the isorropesis, and we have emission. An important fact bearing on the connection between isorropesis and fluorescence has been recorded by Nichols and Merritt;¹ these authors have observed that, when the fluorescence of fluorescein and certain other substances is excited by a beam of ultra-violet light, a distinct absorption occurs of light of the same wave-length as that emitted by the substance when fluorescent.

CONCLUSIONS

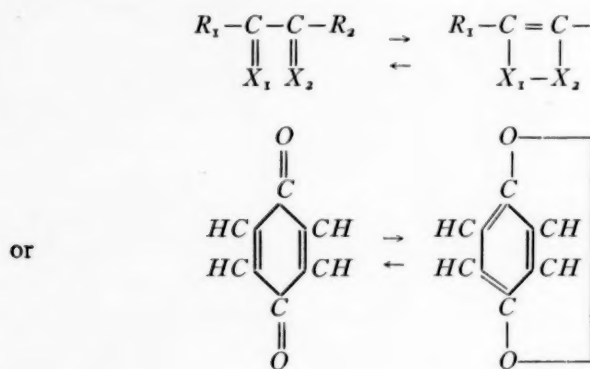
1. When two true ketonic groups $\left(\begin{array}{c} -\text{C}- \\ || \\ \text{O} \end{array} \right)$ are in juxtaposition in the molecule, an oscillation (isorropesis) occurs between the residual affinities of the oxygen atoms, which results in the establishment

¹ *Phys. Rev.*, 18, 447, 1904.

of a free period of vibration in the visible region of the spectrum. These substances are therefore colored.

2. This isorropesis occurs also between the residual affinities of the oxygen atoms in the benzenoid quinones, of the nitrogen atoms of the quinonoid form of the nitroanilines, and of the nitrogen and oxygen atoms of the quinonoid form of the nitrophenols. It also occurs between the residual affinities of the oxygen and nitrogen atoms of the *iso*-nitroso compounds.

3. The process of isorropesis may be expressed chemically by the equilibrium expressed by



4. It is necessary to assume the transitory existence of a meta-quinonoid linking to account for the phenomena observed with meta-nitroaniline and meta-nitrophenol.

5. Many of the physical and chemical properties of benzene are explained by considering that the benzene ring is elastic and undergoes the same vibrations as are suffered by any elastic ring.

6. The meta-quinonoid linking is possible during one phase of the displacement of the benzene ring.

7. In order to start the isorropesis, it is necessary that some influence be present to disturb the residual affinities upon the atoms concerned.

8. This influence is provided in compounds of the type of diacetyl by the neighboring hydrogen atoms which are attracted by the oxygen atoms; in the benzoquinones it is provided both by the hydrogen atoms and also by the benzenoid tautomerism.

9. Subject to the proviso referred to in 7, there is no doubt that this principle may be extended, and that all the phenomena of visible color are due to the oscillation between residual affinities on atoms or groups of atoms in juxtaposition.

10. Any assumption that two compounds must be fundamentally different in constitution if one is colored and the other white is quite untrustworthy.

11. It is possible that color and fluorescence are evidences of the same phenomenon—isorropesis. In the former case the isorropesis provides the mechanism, and the incident light actuates it; in the latter case the isorropesis both provides and actuates the mechanism.

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THE GEOMETRICAL THEORY OF OPTICAL IMAGERY

By JAMES P. C. SOUTHALL

INTRODUCTION

In the admirable preface of Professor S. P. Thompson's translation of Lummer's *Contributions to Photographic Optics* (Macmillan, 1900) he calls attention to the conspicuous superiority of the Germans in all matters relating to the theory and construction of optical instruments, and asserts that, in spite of the existence of some more or less excellent English works on optics—particularly Lord Rayleigh's article in the *Encyclopædia Britannica* and Heath's *Geometrical Optics*—there is nothing in the English language at all comparable, for example, with Dr. S. Czapski's *Theorie der optischen Instrumente nach Abbe* (the first edition of which was published at Breslau in 1893) or the volume on optics written by Dr. Otto Lummer for the ninth edition of Müller-Pouillet's *Lehrbuch der Physik* (Braunschweig, Friedrich Vieweg & Sohn, 1897). Professor Thompson attributes "the badness of almost all recent British textbooks of optics" to the fact that "the examination curse lies over them all;" but, however true this may be, it is only a partial explanation. The secret of the "badness" lies deeper than that. The simple truth of the matter is that neither the English nor any other people have cultivated this domain of mathematical and applied science in a degree at all approaching the methods and activity of the Germans, and in recent years at least all the remarkable developments both in the theory and manufacture of optical instruments are traceable to Germany. To anyone at all familiar with the mathematical investigations of Abbe and Seidel, and the scientific methods of construction employed by such firms as those of Carl Zeiss and Schott & Genossen in Jena, such treatises as Parkinson's *Optics* and Heath's *Geometrical Optics* appear to be more than half a century behind the times.

Of Dr. Czapski's book (of which a new and revised edition has been recently published in Leipzig) it is not too much to say that it has long been recognized as epoch-making; in it was set forth for

the first time a complete and masterly exposition of the remarkable theories of Professor Abbe, of Jena.

One of the particular merits of Abbe's methods consists in the fact that he discerned clearly (what other investigators,¹ including even Moebius and Gauss, appear to have perceived more or less dimly) that the physical agency or mechanism which was employed in the actual formation of an optical image was in no wise involved in the geometrical theory of optical imagery; so that, without any special assumptions whatever as to the construction and mechanism of the optical apparatus, and also without reference to the physical laws of reflection and refraction (which in some form or other always entered into the theories of the earlier investigators), he deduced all the laws concerning the relative positions and dimensions of the object and image, and showed for example, that the so-called "cardinal points," "focal lengths," etc., of the optical system were purely geometrical ideas and essentially independent of the *modus operandi* whereby these conceptions might be said to have a physical existence.

The function of an optical instrument is to produce an image of an external object. Each point of the object is the base (or vertex) of a bundle of rays, of which, in general, only a part is utilized in the formation of the image. These object-rays which are affected by the instrument are called the "incident" rays. Within the apparatus these rays undergo certain refractions (or reflections) at the plane or curved boundary surfaces of suitably disposed optical media; and, thus modified, they "emerge" into the last medium and form somewhere an image of the object, which may be "real" or "virtual," etc.; the nature of the image in the several respects of position, dimensions, and orientation depending primarily on the peculiarity and design of the instrument itself. Proceeding from any point *P* of the object, a bundle of incident rays "enters" the optical instrument, and, emerging therefrom, a portion of these

¹ In this connection see, however, a noteworthy paper, entitled "On the general Laws of Optical Instruments," contributed by J. Clerk Maxwell, in 1858, to the *Quarterly Journal of Pure and Applied Mathematics*; wherein the author expressly discards all assumptions as to the physical mode of producing an optical image, and constructs a purely geometrical theory. In the new edition of his book Czapski calls attention to this paper of Maxwell's, of which, however, Abbe was ignorant.

rays at least, if not all of them, will intersect ("really" or "virtually") in the corresponding (or "conjugate") point P' of the image. In the case of an *ideal* (or geometrically perfect) image, *all* of the emergent rays corresponding to the bundle of incident rays P will intersect in the image-point P' ; so that a bundle of homocentric object-rays will be "imaged" (*abgebildet*) by a homocentric bundle of image-rays.

The fundamental and essential characteristic of optical imagery is, therefore, a point-to-point correspondence, by means of rectilinear rays, between object and image. From this one assumption—at once the most natural and obvious—Abbe deduces all the general laws of optical images. The advantage of this is that in investigating an actual image produced by an optical instrument it will be possible to separate what in the laws of this image depends on the general fundamental laws of optical imagery and what is due to the particular mode of producing the image. Moreover, although today a certain optical instrument may be a mechanical impossibility, we can determine by this theory whether or not it is theoretically practicable, and thus the theory indicates the way of future invention and discovery.

In the modern geometry this unique point-to-point correspondence by means of rectilinear rays between image and object is called "collineation"—a term introduced by Moebius in his *Der barycentrische Calcul* (Leipzig, 1827).

Two regions of space Σ and Σ' are said to be "collinear" with each other if to every point P of Σ there corresponds one (and only one) point P' of Σ' , and to every straight line of Σ which goes through P there corresponds one straight line of Σ' which goes through P' .

In the theory of optics the two regions Σ and Σ' are designated as the "object-space" and "image-space," respectively. Since the relation between the two spaces is perfectly reciprocal, there is no essential difference between them; whence is deduced at once the principle known as "The Reversibility of the Light-Path."

A direct consequence of the unique point-to-point and ray-to-ray correspondence between object-space and image-space is plane-to-plane correspondence; so that to every plane ϵ in the object-space there corresponds also a definite plane ϵ' in the image-space,

and vice versa. This plane-to-plane correspondence, which is likewise characteristic of the collinear relation of two regions of space, is used by Czapski as the basis of his mathematical investigation. Employing the method of analytic geometry, and denoting the co-ordinates of any object-point P , with respect to an arbitrary system of rectangular axes in the object-space, by x, y, z , and the co-ordinates of the conjugate image-point P' , also with respect to an arbitrary system of rectangular axes in the image-space, by x', y', z' , he shows that the following equations, involving fifteen independent constants, are the analytical expression of the relation of collinear correspondence between the object-space and image-space:

$$\left. \begin{aligned} x' &= \frac{a_1x + b_1y + c_1z + d_1}{ax + by + cz + d}, & y' &= \frac{a_2x + b_2y + c_2z + d_2}{ax + by + cz + d}, \\ z' &= \frac{a_3x + b_3y + c_3z + d_3}{ax + by + cz + d}. \end{aligned} \right\} \quad (A)$$

From these general image equations, involving, as we have said, no assumptions as to the apparatus itself, and no restrictions as to the position, form, or dimensions of the object, nor as to the angular aperture of the imaging bundles of rays, all the laws of optical imagery can be deduced.

The geometrical character of the imagery defined by these equations has been extensively investigated in treatises on modern geometry; but, so far as the present writer is aware, there is no book which, *using the methods of projective geometry, treats the theory of collinear correspondence in its special application to the laws of optical imagery.*

Inasmuch as these methods are not only interesting of themselves, but appear in certain respects to possess decided advantages, the writer has proposed to employ them in the following outline of Professor Abbe's theory.

PART I

COLLINEATION OF TWO PLANE FIELDS

ART. I. PROJECTIVE RELATION OF TWO COLLINEAR PLANE FIELDS

For the purposes of this article (which does not pretend to completeness) it will be entirely sufficient and much simpler to restrict

our investigations to two conjugate planes of the object-space and image-space. The advantage of this is obvious, especially as both the methods which we shall use and the results which we shall obtain can be easily extended to include the whole of the two collinear regions of space.

A "plane field" is the name applied to the totality of points and lines contained in a plane.

Definition.—Two plane fields ϵ and ϵ' are said to be "collinear" if to every point P of ϵ there corresponds one point P' of ϵ' and to every straight line p of ϵ which passes through P there corresponds a straight line p' of ϵ' which passes through P' .

Two collinear plane fields ϵ and ϵ' are also called "projective," because to each harmonic range of four points of ϵ there corresponds a harmonic range of four points of ϵ' .

Thus, if P, Q, R, S are a range of four harmonic points of the plane field ϵ , and P', Q', R', S' the four corresponding points of the collinear plane field ϵ' , in the first place, since the points P, Q, R, S , are all situated on a straight line u , the points P', Q', R', S' must all likewise lie on a straight line u' which is conjugate to u . Let $ABCD$ be any quadrangle of the plane field ϵ , such that the two opposite sides, AB and CD , intersect in P , and the other two opposite sides, AD and BC , intersect in Q , while the fifth and sixth sides, BD and AC , go through the points R and S , respectively. To this quadrangle of ϵ there will correspond a certain quadrangle A', B', C', D' of ϵ' , such that the two opposite sides $A'B'$ and $C'D'$, intersect in P' , the other two opposite sides, $A'D'$ and $B'C'$, intersect in Q' , and the fifth and sixth sides, $B'D'$ and $A'C'$, go through the points R' and S' , respectively. Hence, the points P', Q', R', S' are also a harmonic range of points; and this is the condition that the two plane fields ϵ and ϵ' shall be *projective*.

ART. 2. THE SO-CALLED "FLUCHT" POINTS OF CONJUGATE RAYS

Let u and u' denote two conjugate rays of the collinear plane fields ϵ and ϵ' . Since the point-ranges u and u' are "projective" (as has just been shown), it follows that the *anharmonic ratio* ($ABCD$) of any four points A, B, C, D of u is equal to the *anharmonic ratio* ($A'B'C'D'$) of the four corresponding points A', B', C', D' of u' .

That is,

$$\frac{AC}{BC} : \frac{AD}{BD} = \frac{A'C'}{B'C'} : \frac{A'D'}{B'D'}.$$

If the points A, B, C are supposed to be fixed while the point D travels along u , the anharmonic ratio $(ABCD)$ will vary in value; and if the point D moves away to an infinite distance until it coincides with the infinitely distant point E of u , we shall have:

$$(ABCE) = (A'B'C'E') = \frac{AC}{BC},$$

where E' denotes the point on u' which corresponds with the infinitely distant point of u . Since A, B, C are actual (or finite) points of u , no two of which are supposed to be coincident, the value of AC/BC is finite; and, hence, the point E' is a determinate and, in general, a finite point of u' .

Similarly, if F' is the infinitely distant point of u' , the point F , which corresponds to the infinitely distant point F' of u' , is likewise a determinate, and, in general, a finite point of u .

In general, therefore, the points F and E' , corresponding to the infinitely distant points F' and E of u' and u , respectively, are finite (or actual) points having perfectly determinate positions on u and u' , respectively. The German writers call F and E' the *flucht* points of the two projective ranges of points u and u' .

NOTE.—We are careful to say that the so-called “flucht” points are, “in general,” “finite” (or actual) points; for in *one special case*, viz., when

$$(A'B'C'E') = \frac{AC}{BC} = \frac{A'C'}{B'C'},$$

the point E' will coincide with the infinitely distant point F' of u' ; and in this case the infinitely distant points E and F' of the projective point-ranges u and u' will also be conjugate points.

ART. 3. THE “FOCAL LINES” OF CONJUGATE PLANES

In the plane field ϵ let us consider a quadrangle $ABCD$ (Fig. 1) such that the two pairs of opposite sides form two pairs of parallel straight lines. The two parallel sides, AB and CD , intersect in the infinitely distant point P ; and, similarly, the other two parallel sides, AD and BC , intersect in the infinitely distant point Q . If therefore R and S designate the infinitely distant points of the remain-

ing sides, BD and AC , respectively, the four points P, Q, R, S are a harmonic range of points of the infinitely distant straight line e of the plane ϵ .

In the collinear plane field ϵ' the ray $A'B'$, conjugate to AB , will contain the point P' which corresponds to the infinitely distant point P of the ray AB ; so that the point P' is the so-called "flucht" point of the ray $A'B'$. Obviously, the point P' is also the "flucht" point of the ray $C'D'$, conjugate to the ray CD . Precisely in the same way, the point Q' , conjugate to the infinitely distant point Q of the parallel rays AD and BC , is the common "flucht" point of each of the rays $A'D'$ and $B'C'$, conjugate to the rays AD and

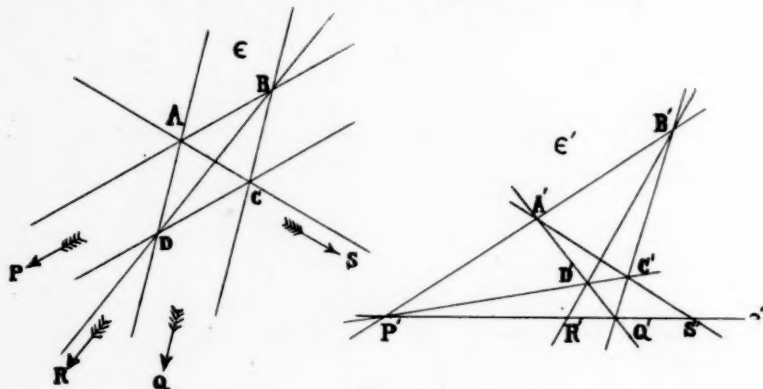


FIG. 1

BC , respectively. Let R' and S' designate the "flucht" points of the rays $B'D'$ (conjugate to BD) and $A'C'$ (conjugate to AC), respectively.

Since (Art. 1) the points P', Q', R', S' are a harmonic range of points, they all lie on a certain definite straight line e' of the plane field ϵ' ; and this line e' , which is conjugate to the infinitely distant straight line e of the plane field ϵ , is the locus of the "flucht" points of all the rays of the plane field ϵ' .

Similarly, there is a certain straight line j of the plane field ϵ , conjugate to the infinitely distant straight line j' of ϵ' , which is the locus of the "flucht" points of all the rays of ϵ .

These two straight lines j and e' are called in the German treatises on geometry the "flucht" lines (or *Gegenaxen*) of the two pro-

jective plane fields. We shall designate them hereafter (from the standpoint of optics) as the *focal lines* of the two conjugate planes ϵ and ϵ' . Hence,

If two plane fields are collinear, then, in general (that is, except in one particular case, to be presently remarked), to the infinitely distant (or ideal) line of one field there corresponds a finite (or actual) line of the other field, the so-called "focal line" of that field.

To a pencil of parallel rays in one plane field there corresponds therefore a pencil of rays in the other field which all intersect in a point situated on the focal line of that field; or, as we might say, the focal line of one plane field is the locus of the bases of pencils of rays which are conjugate to pencils of parallel rays in the other plane field.

The points P', Q', R', S' of ϵ' , corresponding to the infinitely distant points P, Q, R, S , of ϵ , are, in general (as we have said), finite (or actual) points, and determine an actual straight line e' ; except in the one particular case when the quadrangle $A'B'C'D'$ has each pair of its opposite sides parallel. In this special case the points P', Q', R', S' will be ranged along the infinitely distant straight line f' of the plane field ϵ' , and the focal line e' will therefore coincide with the infinitely distant line f' .

This special case in which the two focal lines f and e' are also the infinitely distant lines e and f' of the object-plane and the conjugate image-plane, respectively, is the so-called case of *telescopic imagery* in optics (called "affinity" in modern geometry); of which we shall treat separately in another place. (The name "telescopic imagery" is derived from the fact that the focal points of the optical instrument known as a telescope are both at infinity.)

ART. 4. THE FOCAL POINTS AND THE PRINCIPAL AXES OF THE OBJECT-PLANE (ϵ) AND THE IMAGE-PLANE (ϵ')

As a general thing, therefore, we may say that to a pencil of parallel rays of one plane field there will correspond a pencil of non-parallel rays of the other plane field, the base (or vertex) of which is a finite (or actual) point of the focal line of that plane field. Or, expressing the same fact differently, we can say:

To a parallel (or translatory) displacement of an object-ray there

will (in general) correspond both a translation and a rotation of the conjugate image-ray.

There are two cases, however, in which this is not true:

1. *When the ray is parallel to the focal line.* To the infinitely distant point of the focal line (f) of the object-plane (ϵ), which is the point of intersection of f and the infinitely distant straight line of the plane field ϵ , there corresponds the infinitely distant point of the focal line (e') of the image-plane (ϵ'), which is the point of intersection of e' and the infinitely distant line of the plane field ϵ' . Hence:

To rays of the object-plane which are parallel to the focal line (f) correspond rays of the image-plane which are parallel to the focal line (e'); and vice versa.

In general, this is the only case of a pencil of parallel rays of one plane field corresponding to a pencil of parallel rays of the other plane field. The other case referred to above is:

2. *The case of "telescopic imagery."* In this case the infinitely distant lines of the two plane fields are conjugate to each other and are at the same time the focal lines. Here, therefore, it is obvious that parallel rays of one plane always correspond to parallel rays of the conjugate plane. However, this is a distinctly exceptional case.

A pencil of parallel object-rays will therefore determine a definite point of the focal line (e') of the image-plane; and vice versa. Thus, for example, if we select a pencil of parallel rays of one plane which are all *perpendicular* to the focal line of that plane, we can say:

To a pencil of parallel $\left\{ \begin{matrix} \text{object-rays} \\ \text{image-rays} \end{matrix} \right\}$ which are perpendicular to the focal line $\left\{ \begin{matrix} f \\ e' \end{matrix} \right\}$ of their plane, there corresponds a pencil of non-parallel $\left\{ \begin{matrix} \text{image-rays} \\ \text{object-rays} \end{matrix} \right\}$ which intersect in a certain point $\left\{ \begin{matrix} E' \\ F \end{matrix} \right\}$ of the focal line $\left\{ \begin{matrix} e' \\ f \end{matrix} \right\}$ of the $\left\{ \begin{matrix} \text{image-plane} \\ \text{object-plane} \end{matrix} \right\}$.

This unique pair of points, F and E' , thus determined, are the so called *focal points* of the object-plane and image-plane, respectively.

The two straight lines drawn through F and E' perpendicular to the focal lines f and e' are called the *principal axes* of the object-plane and of the image-plane, respectively. These lines we shall designate by the symbols x and x' , and we shall refer to them as the x -axis and the x' -axis.

Since the ray x' passes through the focal point E' (which corresponds to the infinitely distant point of x) and also through the infinitely distant point of x' (which corresponds to the focal point F , likewise situated on x), it follows that x and x' are a pair of conjugate rays; and *the only pair of conjugate rays which are at right angles to the focal lines.*

METRIC RELATIONS OF TWO COLLINEAR PLANE FIELDS

ART. 5. RELATION BETWEEN CONJUGATE ABSCISSAE

Let u and u' be two conjugate rays of the object-plane and the image-plane, and let R and S' designate the points where these rays cross the focal lines f and e' , respectively. Moreover, let S and R' denote the infinitely distant points of u and u' conjugate to S' and R , respectively. Now, if P, Q and P', Q' are any other pair of conjugate points of u and u' , we shall have:

$$(PQRS) = (P'Q'R'S'),$$

or

$$\frac{PR}{QR} : \frac{PS}{QS} = \frac{P'R'}{Q'R'} : \frac{P'S'}{Q'S'};$$

or, finally,

$$\frac{PR}{QR} = \frac{Q'S'}{P'S'};$$

which may be written:

$$RP \cdot S'P' = RQ \cdot S'Q' = a \text{ constant.}$$

Stated in words, this *characteristic metric relation of optical imagery* may be expressed as follows:

The product of the abscissae¹ of two conjugate points, P and P' ,

¹ The term "abscissa" is employed here, and generally also throughout this paper (unless otherwise specified), to describe the position of a point on a ray with respect to the "flucht" point of the ray as origin. Thus, for example, the abscissa of the point P of the ray u is RP , which means the segment of the ray included between the points R and P , and reckoned from R to P ; that is, reckoned always in the sense in which the letters are written.

In this place we also take occasion to say expressly (although it is doubtless not

of the object-plane and image-plane, with respect to the so-called "flucht" points, R and S' , of two conjugate rays u and u' which go through P and P' , respectively, is constant.

In particular, let us suppose that the two conjugate rays are the two principal axes themselves; then we shall have:

$$FP \cdot E'P' = a \text{ constant.}$$

If we denote the value of this constant for the principal axes (x and x') by a , and if we put

$$FP = x, \quad E'P' = x',$$

we obtain:

$$xx' = a. \quad (1)$$

We shall call this the "abscissa equation."

ART. 6. THE LATERAL MAGNIFICATION

Let m, m' (Fig. 2) denote two conjugate straight lines of the object-plane and image-plane which are perpendicular to the prin-

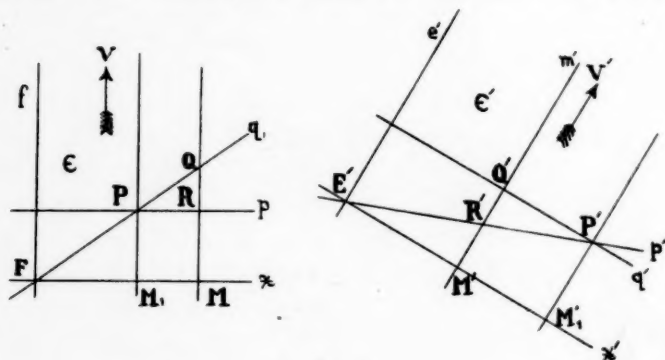


FIG. 2

cipal axes x and x' , respectively, and therefore parallel to the focal lines (see Art. 4). The infinitely distant points, V, V' , of the projective ranges of points m, m' are corresponding points. Let M, M' designate the points of intersection of the lines m, m' with the prin-

necessary) that if A, B, C, D, \dots, J, K are any number of points ranged along a straight line in any order whatever, we have always the following relations:

$$AB + BA = 0; \quad \text{or } AB = -BA;$$

$$AB + BC + CA = 0;$$

$$AB + BC + CD + \dots + JK = AK; \text{ etc.}$$

cipal axes x, x' , respectively. And let Q, Q' and R, R' be any other two pairs of corresponding points of m and m' ; then

$$(QRMV) = (Q'R'M'V');$$

or

$$\frac{QM}{RM} = \frac{Q'M'}{R'M'};$$

which may be written:

$$\frac{M'Q'}{MQ} = \frac{M'R'}{MR} = \beta,$$

where β denotes the value of this constant ratio of corresponding line-segments of the two given conjugate lines m and m' which are parallel to the focal lines f and e' , respectively.

The two conjugate lines m and m' are said to be "projectively similar." This ratio β for each line m of the object-plane perpendicular to the x -axis is called the *lateral magnification* for that line, and has for it a constant value. If, therefore, we put

$$y = MQ, \quad y' = M'Q',$$

we can write:

$$\frac{y'}{y} = \beta;$$

which states that the value of β is independent of the magnitudes of y and y' themselves.

It will be convenient now to select the two focal points F and E' as the origins of two rectangular systems of co-ordinates of the object-plane and image-plane, respectively. The principal axes of the two planes will be the axes of x and x' , while the two focal lines f and e' will be the other two lines of reference (Fig. 2). Let m and m' be two corresponding straight lines at right angles to the axes of x and x' , respectively, and meeting these axes in the points M and M' . And let β denote the value of the lateral magnification for m, m' .

Through any point P of the object-plane draw two rays; viz., the ray p parallel to the x -axis and the ray q which passes through the focal point F . Let R and Q designate the two points on m where the rays p and q , respectively, meet m . On m' lay off $M'R' = \beta \cdot MR$ and $M'Q' = \beta \cdot MQ$; then the intersection of the ray q' drawn through Q' parallel to the x' -axis and the ray p' joining E'

and R' determines the image-point P' corresponding to the object-point P .

If the object-point P is supposed to move along a line perpendicular to the x -axis which cuts this axis in the point M_1 , we see at once that the image-point P' must also traverse a line perpendicular to the x' -axis in such fashion that the lateral magnification $\frac{M_1'P'}{M_1P}$ shall be constant.

Again, if the object-point P is supposed to move along the ray q which passes through the focal point F , the image-point P' will travel along the conjugate ray q' which is parallel to the x' -axis; so that as the ordinate M_1P assumes all values from $-\infty$ to $+\infty$, its image $M_1'P'$ remains constant as to both magnitude and sign.

Thus we see that the lateral magnification β has different values for each pair of conjugate lines which are parallel to the focal lines. That is, the magnification β is a function of the abscissa x .

ART. 7. THE IMAGE EQUATIONS

We proceed, therefore, to ascertain in what way the lateral magnification β depends on x . Let x, y denote the co-ordinates of an object-point Q and x', y' the co-ordinates of the corresponding image-point Q' ; and let $\frac{y'}{y} = \beta$ be the magnification-ratio for the point Q . If θ denotes the angle made with the x -axis by the ray q which joins the focal point F and the object-point Q , then

$$y = x \tan \theta,$$

and

$$\beta = \frac{y'}{x \tan \theta}.$$

If x_1, y_1 and x'_1, y'_1 denote the co-ordinates of the pair of conjugate points P and P' which are situated on the conjugate rays q and q' , and if β_1 denotes the value of the magnification-ratio for the point P , then, since

$$y'_1 = M_1'P' = M'Q' = y',$$

we have:

$$\beta_1 = \frac{y'_1}{x_1 \tan \theta} = \frac{y'}{x \tan \theta}.$$

Therefore,

$$\frac{\beta}{\beta_1} = \frac{x_1}{x}.$$

That is,

$$\beta x = \text{a constant} = b \text{ (say).}$$

Hence, *the lateral magnification β is inversely proportional to the abscissa x .*

Referring, therefore, to (1) of Art. 5, we are able now to express the co-ordinates x' , y' of any point of the image-plane in terms of the co-ordinates x , y of the corresponding point of the object-plane as follows:

$$x' = \frac{a}{x}, \quad y' = \frac{by}{x}. \quad (2)$$

These two equations are the general image-equations for two conjugate planes in their simplest forms.

NOTE.—If, instead of confining the investigation, as we have done here, to two collinear plane fields, we had extended it to two collinear *regions of space*, we should have obtained by precisely analogous processes the following results—which are here given in brief:

1. If we have two collinear regions of space Σ and Σ' , which we shall call in the language of optics the “object-space” and the “image-space,” respectively, then, except in the case of “telescopic imagery” (or “affinity”), to the infinitely distant plane of one region of space there will correspond an actual (or finite) plane, the so-called “flucht” plane or *focal plane*, of the other region of space.

2. In general, to a sheaf of parallel planes of one of the two regions of space there will correspond a sheaf of non-parallel planes of the other region of space. However, there is one exception to this statement: *The two sheaves of parallel planes to which the focal planes themselves belong are conjugate sheaves of planes.*

Two conjugate planes which are parallel to the focal planes are in “affinity” with each other; that is, their infinitely distant straight lines are conjugate to each other. And, hence, any range of points u in the object-space which is parallel to the focal plane of that space will be “imaged” in a projectively similar range of points u' of the image-space which is likewise parallel to the focal plane of that space.

3. To a bundle of parallel rays in the object-space will correspond a bundle of non-parallel rays in the image-space the vertex of which lies in the focal plane of that space; and vice versa. If the bundle of parallel rays in one space meets the focal plane of that space at right angles, the vertex of the corresponding bundle of rays in the other space will determine a certain point in the focal plane of that space—viz., the *focal point* of the object-space or image-space, as the case may be.

The object-space and the image-space have therefore only one pair of conjugate straight lines each of which is perpendicular to the focal plane of the space to which it belongs; these two conjugate lines at right angles to the focal planes are called the *principal axes* of the object-space and image-space, respectively; we shall denote them as the axis of x and the axis of x' .

4. To the sheaf of planes in the object-space which has for its base the x -axis corresponds the sheaf of planes in the image-space which has for its base the x' -axis which is conjugate to the x -axis. Of these so-called "meridian" planes there is one pair at right angles to each other in the object-space to which correspond a pair in the image-space which are also at right angles. Following Czapski, we shall denote these planes as the xy -plane and xz -plane in the object-space and the $x'y'$ -plane and $x'z'$ -plane in the image-space.

Thus, selecting as axes of a system of rectangular co-ordinates the three lines of intersection of the focal plane and the two meridian planes above-mentioned, in the case of both the object-space and the image-space, Czapski shows in his book that the general image-equations (A), which are the analytical expression of the relation of collinear correspondence of two regions of space (see "Introduction") will reduce to the following simple form:

$$x' = \frac{a}{x}, \quad y' = \frac{by}{x}, \quad z' = \frac{cz}{x}, \quad (B)$$

where, instead of fifteen independent constants, the number has been reduced to three (although it is hardly necessary to say that a, b, c of equations (B) do not denote the same constants as those same letters denote in the general equations (A)).

Accordingly, in the most general case of optical imagery, as defined by equations (B), there are involved at least three constants, a, b , and c . In the general case the imagery is not symmetrical around the principal axes of the object-space and image-space; that is, the two magnification-ratios y'/y and z'/z have different values corresponding to the same value of x . In most actual cases, however, the principal axes are axes of symmetry; and as we are concerned primarily with the practical applications of these laws to the theory of optical instruments, we shall assume that this is the case. Thus we shall put $b=c$; in which case the character of the imagery is defined by the two constants a and b , and of the three equations (B) we need deduce only the first two, since the third equation follows at once from the condition of symmetry.

In the case of symmetry with respect to the principal axes of x and x' , it is sufficient, therefore, to ascertain the laws of collinear correspondence of *two conjugate meridian planes* of the object-space and image-space. This is one of the reasons why we have confined the investigation in this paper to the case of two collinear plane fields.

In regard to the constants a and b of equations (2) of Art. 7, it is perhaps not superfluous to say that, in general, the values of these constants are different for different pairs of conjugate planes of the two regions of space. However, for

each pair of conjugate planes of the two sheaves of meridian planes the constants a and b have the same values; and these values are also the same as the values of a and b in equations (B)—assuming that we have symmetry with respect to the principal axes. For any pair of conjugate meridian planes the principal axes of the two planes coincide with the principal axes of the two regions of space.

In the following we shall assume that the two conjugate planes are meridian planes of the object-space and image-space; although generally the results can be applied to any two conjugate planes, as we shall do in certain special cases.

ART. 8. THE FOCAL LENGTHS f AND e'

It was remarked above (Art. 6), and is evident at once from the image-equations, that the lateral magnification $\frac{y'}{y} = \beta$ may have any value from $-\infty$ to $+\infty$, depending on the value of the abscissa x . Accordingly, there must be a pair of conjugate points, H and H' , situated on the axes of x and x' of the object-plane and image-plane, respectively, for which $\beta = +1$. In other words, there is one pair of conjugate straight lines parallel to the focal lines of the object-plane and image-plane, respectively, which are "projectively congruent."

If the two conjugate planes are meridian planes (as we shall now assume), the axes of x and x' are the two principal axes of the object-space and image-space; and the points H and H' are the so-called "principal points" of the optical system—as Gauss designated them in his *Dioptrische Untersuchungen*, published in 1840.

The abscissae of the principal points H and H' with respect to the focal points F and E' are called the focal lengths of the object-space and the image-space, respectively. If we denote the two focal lengths by f and e' , then

$$f = FH, \quad e' = E'H'.$$

If in the image-equations (2) of Art. 7 we put $y = y'$, $x = f$ and $x' = e'$, we find:

$$a = fe', \quad b = f;$$

so that now we may write the image-equations (2) in terms of the two new constants f and e' as follows:

$$\left. \begin{aligned} xx' &= fe', \\ \frac{y'}{y} &= \frac{f}{x} = \frac{x'}{e'} = \beta. \end{aligned} \right\} \quad (3)$$

For general purposes these are the most useful forms of the image-equations.

Czapski defines the focal lengths without reference to the principal points; but our object here was merely to show the final form of the image-equations. It would take too much space to discuss these equations in this place; but the reader who is interested will find in the chapter on the "Geometrical Theory of Optical Images" of Dr. Czapski's book a complete discussion, not only of these equations, but of many other matters connected therewith, which we are compelled to omit, as they are not included within the scope of this article.¹

ART. 9. TELESCOPIC IMAGERY

The image-equations which we have deduced above are not applicable in the special case of telescopic imagery, because in this case the focal lines are the infinitely distant lines of the object-plane and the image-plane.

In the language of geometry, two collinear plane fields ϵ and ϵ' are said to be in "affinity" with each other if their infinitely distant straight lines are corresponding straight lines.

Since, therefore, to each infinitely distant point of one field corresponds an infinitely distant point of the other field, it is obvious that a pencil of parallel image-rays will correspond to a pencil of parallel object-rays. Moreover, to any range of points in one field there will correspond a "projectively similar" range of points of the other field. The peculiarity of such ranges of points is that corresponding segments of them are in a constant ratio to each other.

Let x, x' and y, y' be any two pairs of non-parallel corresponding rays, and let m and n denote the values of the ratios of corresponding segments of each of these two pairs of conjugate rays, respectively. Let PQ and $P'Q'$ be two conjugate line-segments drawn parallel to x and x' , respectively. If then we draw through the points P and Q lines parallel to y , the corresponding lines in the other plane-

¹ The reader is also referred to E. Wandersleb's article in the first volume of *Die Theorie der optischen Instrumente*, edited by M. von Rohr (Berlin, 1904). This valuable and exhaustive work, of which only the first volume has appeared, is published under the auspices of Dr. Czapski himself, who contributes the preface. Each chapter is the work of one or more of the staff of eminent optical engineers associated with the firm of Carl Zeiss, in Jena.

field will be a pair of lines drawn through the points P' and Q' parallel to y' , and we see at a glance that

$$\frac{P'Q'}{PQ} = m.$$

Hence, all lines parallel to x have the same magnification-ratio. Thus, we can say:

In the case of telescopic imagery, all parallel rays of the object-plane have the same value for the magnification-ratio.

If we select any two conjugate points, O and O' , as origins of the axes of co-ordinates of the object-plane and the image-plane, there will always be in the two projective pencils of rays through O and O' one pair of rectangular rays in the object-plane which correspond to a pair of rectangular rays in the image-plane. Hence, in the case of telescopic imagery, the image-equations referred to rectangular axes may be written as follows:

$$x' = mx, \quad y' = ny. \quad (4)$$

ART. 10. IDEAL IMAGE NOT ATTAINABLE

The geometrical theory of optical imagery is seen, therefore, to be identical with the theory of collineation in modern geometry. As to the mechanical means of producing an optical image we have not inquired at all in the foregoing; although, properly speaking, the geometrical theory is at the foundation of the theory of all optical instruments, and a knowledge of the theory has proved of the utmost advantage in the design and construction of optical apparatus.

In general, there may be said to be only one actual optical system which perfectly satisfies the condition of collinear correspondence—viz., the *plane mirror*; which, inasmuch as it produces only a virtual image without magnification, hardly deserves to be ranked as an optical instrument at all. The *pin-hole camera* is no exception to the statement just made, because only when the aperture is a mathematical point would there be strict point-to-point correspondence of object and image—even then assuming that there were no exceptions to the law of the rectilinear propagation of light, such as we encounter in physical optics.

Practically speaking, nearly all optical instruments consist of combinations of certain isotropic optical media separated from each

other by spherical boundary-surfaces, the centers of which are ranged along a straight line, called the "optical axis" of the system. This optical axis coincides with the principal axes of the object-space and image-space, and is an axis of symmetry. An optical system of this kind is called a "centered" system of spherical surfaces. (In this statement it seems superfluous to add that plane boundary-surfaces are likewise included, inasmuch as they may be regarded as spherical surfaces with infinite radius.) By means of an optical apparatus of this description it is practically quite impossible to produce, with the employment of wide-angle bundles of rays, a theoretically perfect image of an extended object; that is, a homocentric bundle of rays after passing through the instrument is, in general, no longer homocentric. Instead of the ideal case of collinear correspondence of object-space and image-space, the theory of optical instruments is complicated by numerous practical and irreconcilable difficulties due chiefly to the so called "*aberrations*"—a number of which are aberrations of sphericity, while others are "chromatic" aberrations. Nevertheless, in spite of these apparently insurmountable obstacles in the way of the attainment of even an approximately perfect image, the wonderful performance of modern optical apparatus is a most extraordinary and convincing proof, not only of the genius and skill of the designers of these instruments, but of the achievements that are possible under science as the leader. However, it would be foreign to the purpose of this paper to do more than merely refer to the intricate problems of applied optics.

The point in which the optical axis of a centered system meets one of the spherical surfaces is called the "vertex" of that surface. If we render opaque all the parts of these spherical "interfaces" (or boundary-surfaces) except the very small zones of which these vertices are the summits, the only rays of light that can be utilized by the optical instrument are the narrow bundles of rays which meet the spherical surfaces at very nearly normal incidence. These rays which are all very near to the optical axis are called *paraxial rays*. *In the case of the refraction (or reflection) of paraxial rays by any optical system consisting of a series of centered spherical surfaces, there is strict collinear correspondence between object and image*

for "monochromatic" rays of light. All of the formulæ, therefore, which we have obtained are applicable in the case of paraxial rays. These formulæ were obtained for paraxial rays by the earlier investigators in optics, chief among whom may be mentioned Mœbius, Gauss, Bessel, and Listing, who made great advances in the theory of optics, so that they may be regarded as the founders of modern optics. The first writers on optics (such as Kaestner, Euler, Kluegel, Herschel, and Littrow) did not employ general formulæ and calculated the paths of the rays from surface to surface without simplifying the problem by introducing the ideas of the so-called "cardinal points" of the optical system.

PART II

THE ELEMENTARY PROBLEMS OF GEOMETRICAL OPTICS

Most textbooks of geometrical optics begin with the problems of reflection and refraction of paraxial rays at a spherical surface, and proceed thence to the problem of infinitely thin lenses. As already stated, we have always in the case of paraxial rays complete collinear correspondence between object-space and image-space. Moreover, in each of the problems above mentioned we have a simple case known in geometry as "central collineation." Inasmuch as these problems afford pretty applications of the general theory outlined in the preceding pages, and are at the same time not without interest for their own sake, a consideration of them here should not be deemed out of place. Perhaps, too, to teachers of physics, especially, as the author ventures to hope, this mode of viewing these problems may be in some respects novel and suggestive.

ART. II. CENTRAL COLLINEATION OF TWO PLANE FIELDS

If two collinear plane fields are so situated relative to each other that they have *in common a self-corresponding range of points*, we have the case of *central collineation* of two plane fields. The straight line common to the two fields, which corresponds with itself point by point, is called the "*axis of collineation*." Obviously, any pair of corresponding rays of the two collinear plane fields will meet in this line. If the two plane fields are not in the same plane, the axis of collineation will be the line of intersection of their planes.

In the problems in optics, however, the two plane fields are usually in the same plane of space. We shall assume here that this is the case, and also that this plane is a meridian plane; that is, a plane which contains the optical axis of the system.

It is easy to show that *in the case of central collineation of two plane fields the straight lines which join each pair of conjugate points all intersect in one point.*

This point which is situated on every straight line which joins a pair of conjugate points is called the *center of collineation*.

By way of illustration of this kind of collineation, we may consider the following problem:

Given the axis of collineation (y) and the center of collineation (C) and the positions of two conjugate points P and P' : it is required to construct the image-point Q' of a given object-point Q (Fig. 3).

Through the object-points P and Q draw the straight line k meeting the axis of collineation (y) in the self-corresponding point K ; the corresponding straight line k' which connects K with P' must also pass through the image-point Q' . And since Q' must likewise be situated on the self-corresponding ray which passes through Q and the center C , the image-point Q' will be uniquely determined by the intersection of the straight lines KP' and QC .

Similarly, the image-ray l corresponding to any object-ray l' may be constructed as follows: Let L designate the point where l meets the axis of collineation (y), and let G be the point of intersection of l and any ray k which passes through the given object-point P . The image of G is the point of intersection of the straight lines joining C with G and K with P' . Hence, the image-ray l' is the straight line joining G' and L .

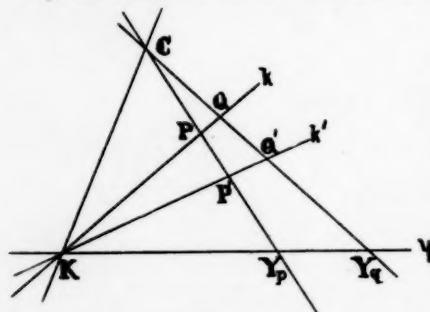


FIG. 3

The self-corresponding ray at right angles to the axis of collineation (y) is the optical axis of the system. This ray we shall refer to as the ray x of the object-space and the ray x' of the image-space. The point where it crosses the axis of collineation will be desig-

nated by the letter H , and this point will be selected, in the special case of central collineation, as the most convenient point for the origin of a system of rectangular co-ordinates the axes of which are the optical axis and the axis of collineation.

A good illustration of central collineation is afforded by the *refraction of paraxial rays at a spherical surface* (Fig. 4). The center of the spherical surface is evidently the center of collineation (C), because the rays which pass through this point (really or virtually) meet the refracting surface normally and proceed in unchanged directions. Moreover, since we are concerned only with

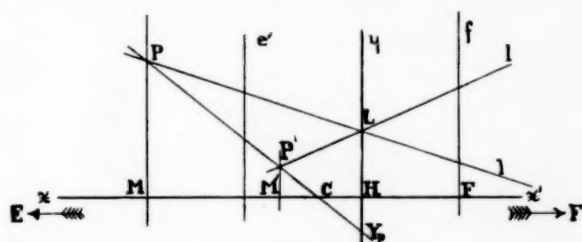


FIG. 4.—On the assumption that light is propagated from left to right, this diagram shows imagery by refraction of paraxial rays from air to glass at concave spherical surface.

paraxial rays which meet the refracting surface at points infinitely near to the vertex (H), it is plain that the straight line in the meridian plane which is *tangent to the spherical surface at its vertex* is the axis of collineation (y).

ART. 12. THE FOCAL POINTS

It is obvious that the *focal lines*, f and e' , of the object-plane and image-plane are both parallel to the axis of collineation (y); because each focal line meets its corresponding straight line on the axis of collineation and at the same time in an infinitely distant point. The *focal points* are the points, designated by F and E' , where the optical axis meets the focal lines. Now, if E and F' are the infinitely distant points of x and x' —that is, if E and F' are the points conjugate to the focal points E' and F , respectively—we can write the following equation immediately:

$$(CFHE) = (CF'HE');$$

that is,

$$FH = CE'. \quad (5)$$

Hence, also:

$$E'H = CF. \quad (6)$$

The important relation expressed by these equations is characteristic of central collineation; it may be stated in words as follows:

In the case of central collineation, the two focal points are always so situated that the step from one of them to the point H is identical with the step from C to the other focal point.

(The reader will find this a very useful thing to remember in connection with problems of refraction of paraxial rays at a spherical surface.)

If we denote the abscissae of the center of collineation (C) and the two focal points (F and E') by the symbols r , s_F and s'_E ; that is, if we put

$$HC = r, \quad HF = s_F, \quad HE' = s'_E,$$

then since

$$FH = CE' = CH + HE',$$

we obtain:

$$s_F + s'_E = r. \quad (7)$$

That is, *the sum of the abscissae of the focal points is equal to the abscissa of the center of collineation.*

The abscissae s_F and s'_E are not the focal lengths; if f and e' denote the focal lengths of the optical system, then $f = FH$, $e' = E'H$, so that

$$s_F = -f, \quad s'_E = -e'.$$

If therefore we are given the center of collineation (C) and the axis of collineation (y), together with (say) the ratio of the two focal lengths of the optical system, we can construct the two focal lines and determine completely the nature of the imagery.

ART. 13. THE "INVARIANT" IN THE CASE OF CENTRAL COLLINEATION

Since all the rays of the pencil C are self-corresponding, each of these rays is the base of two projective ranges of points, a range of object-points and a range of corresponding image-points. Moreover, each of these self-corresponding rays has two self-corresponding points, of which one is the center of collineation itself, and the other is the point where the ray crosses the axis of collineation.

Similarly, every point on the axis of collineation is the common vertex of two projective pencils of rays, a pencil of object-rays and a pencil of corresponding image-rays. Each pair of such pencils of rays contains two self-corresponding rays, of which the axis of collineation itself is one, and the ray joining the common vertex of the two pencils with the center of collineation is the other.

If P, P' (Fig. 3) are two conjugate points, and if the line joining P and P' crosses the axis of collineation in the point Y_p , the anharmonic ratio of the two conjugate points P, P' with the two self-corresponding points C, Y_p of the two projective ranges of points which have the common base PP' has a fixed value c which is independent of the positions of the conjugate points P and P' ; that is,

$$(CY_pPP') = c.$$

This is an elementary proposition of projective geometry. If Q, Q' are any other pair of conjugate points, and if Y_q designates the point where the ray QQ' crosses the axis of collineation, and, finally, if S is the point on the axis of collineation where the two corresponding rays PQ and $P'Q'$ intersect, it is obvious at once that the range of points C, Y_p, P, P' is in perspective with the range C, Y_q, Q, Q' , since they are sections of the pencil of rays S . And, hence, the anharmonic ratios of each of these ranges of four points are equal. Accordingly, we have the following remarkable relations:

$$\left. \begin{aligned} c &= (CY_pPP') = (CY_qQQ') = \text{etc.}, \\ &= (CHFF') = \frac{CF}{HF}, \\ &= (CHEE') = \frac{HE'}{CE'}; \end{aligned} \right\} \quad (8)$$

where, as heretofore, F and E' are the two focal points and F' and E the corresponding infinitely distant points (see Fig. 4).

The most striking characteristic of central collineation, therefore, consists in the fact, above discovered, that it has an *invariant*, which is the constant anharmonic ratio of any two corresponding points with the two self-corresponding points of the ray connecting the corresponding points. The value of this invariant, as above stated, is:

$$c = \frac{CF}{HF} = \frac{HE'}{CE'}.$$

Using the same symbols as in Art. 12 to denote the abscissae, with respect to H , of the axial points C , F , and E' , we obtain:

$$\frac{s_F - r}{s_F} = \frac{s'_E}{s'_E - r} = c;$$

which gives the relation obtained in Art. 12, viz., $s_F + s'_E = r$; accordingly we have:

$$\frac{s'_E}{s_F} = -c. \quad (9)$$

Thus we ascertain that *the value of the invariant (c) is equal to the ratio of the abscissae of the focal points, with the sign changed*. (We may likewise write $e'/f = -c$, since the focal lengths have the same ratio as the abscissae of the focal points).

In particular, if M , M' designate the positions of any two conjugate axial points (that is, conjugate points situated on the optical axis), then

$$(CHMM') = c,$$

or

$$\frac{CM}{HM} : \frac{CM'}{HM'} = c.$$

We shall denote the abscissae of M , M' , with respect to H , by the symbols s , s' , respectively; that is,

$$HM = s; \quad HM' = s', \quad \text{also, } HC = r.$$

Substituting these symbols in the above, we obtain the following relation between the abscissae of any pair of conjugate points *for any central collineation*:

$$\frac{c}{s'} - \frac{1}{s} = \frac{c-1}{r}. \quad (10)$$

The applications of this formula will be shown in the following section.

ART. 14. THE CASES WHICH WE HAVE IN OPTICS

If the *sign of the invariant (c) is positive*, the conjugate points M , M' are not "separated" (in the geometrical sense) by the axis of collineation (y) and the center of collineation (C). That is, for $c > 0$, the points M and M' are either both situated between C and H , or neither of them is between C and H . This case occurs always whenever light is *refracted* from one medium into another; so that

in optics a positive value of c denotes refraction; whereas, on the contrary, whenever the light-rays are *reflected* at a mirror, the imagery is of the kind that corresponds to a *negative* value of c ($c < 0$); in which case one of the points M or M' will lie between C and H , but not the other point.

I. REFRACTION OF PARAXIAL RAYS; $c > 0$

1. Suppose that r is *not equal to zero*; that is, suppose that the center of collineation (C) does not lie on the axis of collineation (y). In this case formula (10) is the formula for the *refraction of paraxial rays at a spherical surface* (Fig. 4). The invariant (c) in this case proves to be identical with the "relative index of refraction" (n) from the "first medium" to the "second medium," while the other constant (r) denotes here the radius of the spherical surface. Accordingly, the points H and C designate the positions of the "vertex" and center of the spherical surface, respectively. If we suppose in our diagrams that the light is represented as being propagated from left to right, then, according as the center C is to the right or left of the vertex H , the spherical refracting surface will be "convex" or "concave." If, therefore, we choose the direction of propagation of light along any ray as the *positive direction of the ray*, a convex surface will be indicated by a positive value of r . Writing n in place of c in the general formula (10), we obtain the special formula for the refraction of paraxial rays at a spherical surface in the usual form given in the textbooks, viz.:

$$\frac{n}{s'} - \frac{1}{s} = \frac{n-1}{r};$$

There are several special cases under this head which may be noticed in passing:

a) Suppose that $c = +1$ (the value of r , as above specified, being different from zero). Now the value of the refractive index is unity ($n = c = +1$), and the equation above gives $s = s'$. Hence, object-space and image-space coincide point by point; in fact, the two spaces are identical. When $n = +1$, there is no optical difference between the first medium and the second.

b) *The case when $r = \infty$.*—This merely means that the center C is at an infinite distance away in the direction of a line at right

angles to the axis of collineation; so that now the refracting surface is a *plane* surface. The formula in this case becomes:

$$s' = ns,$$

which is, therefore, the abscissa-relation in the case of the *refraction of paraxial rays at a plane surface*. Since the center of collineation (C) is at an infinite distance in a direction perpendicular to the axis of collineation (y), all straight lines which join pairs of conjugate points are parallel to the abscissa-axis (Fig. 5). The two infinitely distant straight lines of the two collinear plane fields must both pass through the point C , and therefore they must be self-corresponding rays. Consequently, this is a case of *telescopic imagery*, for which the lateral magnification $y'/y = +1$ (see Art. 9).

2. The invariant $c = +1$ and $r = 0$.—If $r = 0$, the center of collineation (C) is situated on the axis of collineation (y). Putting $r = 0$ in equation (7), we find:

$$s_F + s'_E = 0;$$

so that this type of imagery is characterized by the fact that

the two focal points are equidistant from the axis of collineation (y), and on opposite sides thereof. This will be immediately identified as the imagery produced by the *refraction of paraxial rays through an infinitely thin lens*. In the diagram (Fig. 6) the lens itself is represented by the axis of collineation (y), and the point H which coincides with the center of collineation is the "optical center" of the lens. The anharmonic ratio $(CHMM') = +1$ merely states that the points C and H are coincident—that is, $r = 0$, as we know already. Formula (10) is therefore of no value here, but we can obtain the so-called "lens formula" by writing:

$$(HMF'E) = (HM'F'E'),$$

where M, M' denote any pair of conjugate points on the optical axis and E and F' denote the infinitely distant points corresponding to the focal points E' and F , respectively. Hence, we have:

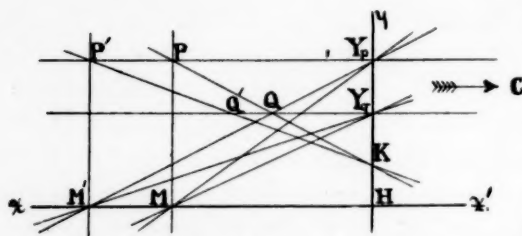


FIG. 5.—Refraction of paraxial rays at plane surface.

$$\frac{HF}{MF} = \frac{M'E'}{HE'}$$

and, since here $HF + HE' = 0$, that is,

$$HE' = FH,$$

we obtain:

$$\frac{HF}{MF} = \frac{M'E'}{FH},$$

which gives:

$$FM \cdot E'M' = -FH^2;$$

so that, if we put $FM = x$, $E'M' = x'$, and $FH = f$, we can write:

$$x \cdot x' = -f^2;$$

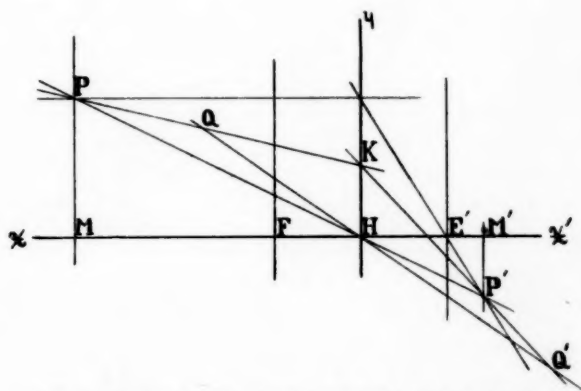


FIG. 6.—Refraction of paraxial rays by infinitely thin lens.

which is a useful form of the *lens formula*. Or, putting $HM = s$, $HM' = s'$, so that we have:

$$x = s + f, \quad x' = s' - f,$$

and substituting these values of x and x' in the above equation, we shall derive the lens formula in its usual form:

$$\frac{1}{s'} - \frac{1}{s} = \frac{1}{f}.$$

(If we take a point L on the optical axis such that $HL = 2HM = 2s$, this equation shows that we have $(LHFM') = -1$; that is, the points L and H are harmonically separated by F and M' ; which suggests an easy geometrical way of constructing the image-point M' corresponding to the object-point M .)

The same formula is applicable also to the refraction of paraxial rays through any number of thin lenses in contact with each other; in this case f denotes the focal length of the combination of lenses.

II. REFLECTION OF PARAXIAL RAYS; $c < 0$

The only negative value of the invariant (c) that has any practical meaning in optics is the value $c = -1$. For this value we have:

$$(CHMM') = -1;$$

so that each pair of conjugate points is harmonically separated by the center (C) and the axis of collineation (y). The general formula (10) becomes for this case:

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{r},$$

which is seen to be the formula for the *reflection of paraxial rays at a spherical mirror*. Putting $c = -1$ in (9), we find $s_F = s'_E$; hence, in the case of a spherical mirror the focal points F and E' are coincident. The fact that the points C, H, M, M' are a harmonic range of points affords an easy method of constructing the image-point M' of a given object-point M .

If $r = \infty$, we have $s = -s'$ (case of *reflection at a plane mirror*).

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PRELIMINARY PAPER ON THE CAUSE OF THE CHARACTERISTIC PHENOMENA OF SUN-SPOT SPECTRA¹

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In considering the characteristic features of the spectra of sun-spots, three points especially attract attention:

1. The fact that certain lines in the spectrum of a given element are strengthened, while others are weakened; the remainder of the lines being unaffected.
2. The inclusion of all the strengthened lines within the visible spectrum, none of them occurring in the ultra-violet, and their predominance in the red, yellow, and green.
3. The relatively great intensity of the continuous background² of the spot spectrum in the less refrangible region.

From our general knowledge of spectra corresponding to various temperatures we are aware:

1. That in passing from a high temperature to a lower temperature, certain lines are relatively strengthened, some are unaffected, and others are diminished in intensity.
2. That such a reduction of temperature is accompanied by an increase in the relative intensity of the less refrangible lines, and a shift of the maximum of a continuous spectrum toward the red.

The general correspondence of these two groups of facts led us to seek for an explanation of the spectra of sun-spots, on the hypothe-

¹ *Contributions from the Solar Observatory*, No. 11.

² Careful visual observations of the spot spectrum have been made with the Littrow spectroscope of 18 feet (5.5 m) focal length in the third and fourth orders of a 4-inch (10 cm) grating, having 14,438 lines to the inch (5,672 to the cm). Although an immense number of fine lines can be seen in the spot spectrum, they nevertheless seem to lie on a continuous dark background, which we have not been able to resolve into lines. This background, however, is interrupted at certain points by lines or breaks, which seem to be nearly as bright as the spectrum of the adjacent photosphere. They do not appear to us like genuine bright lines, and we are unable to offer an adequate explanation of them, unless the dark background is resolvable. Up to the present time we have observed none of the bright reversals (narrow bright line on wider dark line) of spot lines described by Mitchell, although the high resolving power of our spectroscope should render such reversals easily visible.

sis that the metallic vapors within the spots have a temperature lower than that of the photosphere.

Fortunately, the best of material was available for the investigation. Photographs of spot spectra, made with the Snow telescope and the Littrow spectrograph of 18 feet (5.5 m) focal length, show an immense number of affected lines.¹ These plates, which cover the region from D to $H\beta$, have been supplemented by photographs of the entire spectrum of recent large spots, extending from A in the red to the ultra-violet, made by Mr. Ellerman. Plates extending into the ultra-violet, which had previously been used for a study of the H and K lines, were also available.

The laboratory work began with a study of iron and other metals in a synchronous rotating arc, designed by Professor Crew, and constructed for us under his supervision. This ingenious instrument permits the spectrum of the alternating arc to be photographed at any desired phase. In an account of his experiments with the arc, published last year in the *Astrophysical Journal*, Professor Crew showed that the changes in the relative intensities of lines photographed at phase angles varying from 90° to 0° correspond to the changes observed in passing from high to low temperatures. The arc gives excellent results, but the brightness decreases rapidly with the phase, involving an undesirable increase of exposure time. It therefore occurred to Mr. Gale to try the effect of varying the current strength in an ordinary 110-volt direct current arc, the difference of potential between the poles being kept approximately constant. Spectra photographed with currents of 30 amperes and 2 amperes respectively show changes of intensity similar to those observed with the synchronous arc, with the advantage that the spectrum given by the arc with low current requires a much shorter exposure than that of the low-phase synchronous arc, which is necessarily intermittent. We have thus photographed, with currents of 30 amperes and 2 amperes, the spectra of iron, titanium, vanadium, chromium, manganese, calcium, and other metals characteristic of sun-spots. As the work progressed a correspondence was observed between lines that are "enhanced" in the spark and those that are weakened in sun-spots. For the further study of this effect we have also photo-

¹ For a partial list of the strengthened lines see *Astrophysical Journal*, 23, 11, 1906.

graphed the spectra of the same elements in the discharge of a 600-watt transformer, giving about 6,000 volts at the secondary terminals. A condenser was used in the discharge circuit, and the potential was increased by an auxiliary air spark, in series with the observed spark, both being exposed to a strong blast of air from an electric fan. Under these conditions the "enhanced"¹ lines of the spark are well shown in the photographs. We are about to investigate them more effectively with the aid of a 5 K. W. transformer, giving 1,000, 2,000, 4,000, 8,000, 16,000, 32,000, or 64,000 volts, as desired.

The instrument with which by far the greater part of the laboratory spectra used in this investigation have been secured is a grating spectroscope in the Littrow form of 13 feet (3.96 m) focal length. The grating is by Michelson, with 700 lines to the millimeter, and gives a bright first-order spectrum on one side with excellent definition. The slit is provided with an occulting bar, by means of which two spectra that are to be compared can be photographed in the usual way. In most of this work, however, we have preferred to make the separate exposures through the same window in the bar, moving the plate between exposures. In this way we have been able to obtain a greater range of exposure time, and the arrangement has proved satisfactory, since the slight displacements between the separate spectra are of no consequence for our purpose. The window used has a diameter of about 1.2 mm. Our usual procedure in comparing the weak and the strong arcs has been to place on each side of the weak arc spectrum two spectra of the strong arc, giving different exposure times in the two cases. From the four strong arc exposures obtained in this way that one is selected which is most nearly comparable in general strength with the weak arc spectrum. When the spark spectrum has also been added, it has usually been placed immediately adjoining the weak arc, with the arrangement of the strong arc spectra as before. A few plates have been taken with a Fuess quartz spectrograph, but these have been used exclusively for qualitative purposes.

The arc employed in the work is of the self-feeding type, with carbon poles, working on a direct current (storage battery) circuit

¹ Throughout this paper the word "enhanced" is used to denote lines that are strengthened in the spark as compared with the arc.

of 110 volts. The metal has in all cases been placed in the positive pole.

EXPLANATION OF THE TABLES

The tables which follow contain the results of a study of the elements titanium, vanadium, iron, chromium, and manganese, for a region extending from the ultra-violet to λ 5800. Our investigation of both spot and laboratory spectra is far from complete, but the material here presented seems sufficient for the purpose of a preliminary communication. The tables include all the lines which are affected prominently, and which, through rise or fall in intensity in spot as compared with disk, or in spark or weak arc as compared with strong arc, are especially significant in the present investigation. Until it becomes possible, by the use of improved experimental methods, to increase the magnitude of the effects observed in the laboratory, it is necessary to omit from the discussion such slight apparent changes as are now beyond the limits of accurate observation.

In a discussion of the behavior of the lines of these elements, the investigation naturally divides itself into two parts: the relation of the weak arc to the strong arc, and of the arc to the spark. Accordingly, two sets of tables are given. The first contains the wavelengths of all the lines which are affected prominently in spots, the amounts by which they are affected, their behavior in the weak arc as compared with the strong arc, and in the spark as compared with the weak arc. For these last two determinations a scale of 0 to 5 has been used, 5 denoting the maximum change. In the column "Identification, Rowland," the absence of an entry indicates that in the Preliminary Table the line is identified with the element considered, a dash that the line is unidentified, while identifications with other lines are given in full. As comparatively few of the fainter lines given in Rowland's table are identified by him, it has been necessary to obtain such identifications for a considerable number of lines, particularly of vanadium and titanium. For the green and yellow region of the spectrum these have been made from measures of plates taken in our laboratory, further results of which will be published at a later time. Excellent tables by Hasselberg of *Ti*, *V*, *Mn*, and *Cr*, and by Rowland and Harrison of *V*, are available, and have been used for the blue and violet region of the spectrum. The

TABLE I
SUN-SPOT LINES IN SPARK AND ARC
TITANIUM

A Rowland	Identification Rowland	Solar Intensity	Spot Intensity	Amount Increase Weak Arc	Amount Enhance- ment Spark	Remarks
4060.42	<i>Fe, Ti</i>	1	2	1		
4112.87		1	2	0-1		
4299.80		2	3	n.c.		
4300.21		3	2	-2	4	
4300.73	<i>Ti?</i>	2	3	1		
4338.08		4	3	-2	3	
4351.00		1	0	-2	2	
4387.01		1	0	-1	3	
4426.20	<i>oNd?</i>	2	2	1		
4471.41		0	2	1		
4475.03		0	1-2	n.c.		
4496.32		1	2	0-1		
4501.44	<i>Ti, —</i>	5	4	-2	4	
4512.91		3	4-5	1		
4518.20		3	4-5	1		
4522.97		2	4	1		
4533.42	<i>Ti—</i>	4	5-6	0-1		
4534.95		4	5	1		
4548.94		2	3	0-1		
4555.66		3	4	0-1		
4572.16	<i>Cr</i>	6	5	-2	5	
4617.45		3	4	1		
4623.28		2	3	0-1		
4639.54		2	4	1		
4639.68	<i>Ti, La</i>	0	3	1		
4639.85		2	3	1		
4640.12		1	3-4	1		
4645.37		0	1-2	n.c.		
4656.64	<i>Ti, La</i>	3	5	1-2		
4682.09		3	5	1-2		
4693.85		0	1-2	0-1		
4722.80		0	2	0-1		
4742.98	<i>Ti, La</i>	1	2	0-1		
4758.31		1	2	1		
4799.98		1	2	0-1		
4820.59		1	3	1		
4841.07	<i>Ti, La</i>	3	4	1-2		
4856.20		1	3	1		
4868.45		0	1-2	0-1		
4870.32		1	2	1		
4885.26	<i>Ti, La</i>	2	3	1		
4900.10		2	3	1		
4913.80		2	3	1		
4928.51		0	1-2	1		
4981.91	<i>Ti, La</i>	4	5	1-2		
4991.25		3	4-5	1-2		
4997.28		0	2	1		
4999.69		3	4-5	2		

TABLE I—Continued

A Rowland	Identification Rowland	Solar Intensity	Spot Intensity	Amount Increase Weak Arc	Amount Enhance- ment Spark	Remarks
5009.83	<i>Ti, Co</i>	00	2-3	1-2		
5016.34		2	4	1		
5020.21		2	3-4	1-2		
5023.05		2	3	1		
5025.03		3	4-5	1		
5025.75		1	2	1		
5036.64		2	4	1-2		
5038.58		2	3	1		
5040.14		3	4	1-2		
5064.84		3	4-5	2		
5087.24		0	2	1-2		
5147.65		0	2	1-2		
5173.92		2	3	1-2		
5186.07		2	1	-1	2	
5193.14		2	3	2		
5210.56		3	4-5	2		
5219.88		0	3	1		
5238.74		000N	2	1-2		
5266.14		0	1	1-2		
5282.58		00	1	0-1		
5336.97	<i>Ti, —</i>	4	3	-2	3	
5426.47	—	00	3	1-2		H.
5460.72	—	00	3	1		H.
5471.41	<i>Ti?</i>	000	2	1-2		
5474.44		00	1	1		
5477.90	—	00	1-2	1		
5482.08		00	1-2	0-1		H.
5490.37	—	0	3	1-2		
5490.90	—	000 }	2	n.c.		H.
5491.04	—					
5504.12	—	0	1	1		
5512.74		2	4	2		
5514.56		2	3-4	2		
5514.75		2	3	2		
5523.47		000 }	0	1		
5523.55		000 }				
5565.70		00	2	1-2		
5644.26		00 }	2-3	2		
5644.37		0 }				
5648.80		00	2	1		
5662.37		0	2-3	1		
5680.15		000	0	1		H.
5702.88		000	0-1	1-2		
5716.67		00	2	1-2		
5720.67	<i>Ti, A</i>	0	2	1		
5739.70	—	0	1-2	0-1		H.
5740.20	—	0	1	0-1		H.
5766.55	<i>Ti, A</i> <i>Ti, Cr</i>	0	1-2	1		
5774.25		0	1	1		
5786.19		0N	2	0-1		
5804.48		0	1-2	1		
5866.68		3	5	2		

TABLE I—Continued

CHROMIUM

λ Rowland	Identification Kowland	Solar Intensity	Spot Intensity	Amount Increase Weak Arc	Amount Enhance- ment Spark	Remarks
4339.62		4	5	2		
4351.22		3	4	2		
4371.44		2	3-4	1-2		
4373.42		1	3	1		
4540.67		2	3	1		
4555.16	—	2	1	-1	1	L.
4558.83	Cr?	3	2	-1	4	
4571.85		1	1-2	n.c.		
4580.23		3	4-5	1		
4588.38	—	3	2	-1	3	H., L.
4600.93		3	5	2		
4616.30		4	6	2		
4618.07	Fe—	4d?	3		1-2	L.
4626.30		5	7	1-2		
4646.35		5	6	2		
4651.46		4	6	1		
4652.34		5	7	2		
4756.30		2	3-4	1-2		
4789.53		2	3	1-2		
4824.32	Fe	3	2	-1	4	L.
4829.55		2	3	1		
4836.42	—	0	00		1	L.
4848.44	—	2	1		3	H., L.
4862.03		0	2	1		
4887.19	Ni, Cr	2	2-3	1		
4936.51		1	2	0-1		
4942.66		2	3	1		
5073.11		1	2	1		
5123.64		000	0	1		
5144.85	Cr, C	00	1	0-1		
5166.45	Cr-Fe	3	4	1		
5227.04	Fe-Cr	3	3-4	1		
5230.38	Co, Cr	00	0-1	0-1		
5237.49	Cr?	1	0-1		3	
5239.14		00	1	n.c.		
5243.53		00	0-1	0-1		
5247.74		2	3-4	2		
5264.33		4	10	2		
.42	Ca	3				
5275.93		1	1-2	1		
5296.87		3	5	1-2		
5298.46		4	6	2		
5300.93		2	3	1-2		
5335.05	Co	1	0		2	
5345.99		5	6-7	2		
5348.51		4	5	1-2		
5410.00		4	6-7	3		
5442.63		00	0-1	n.c.		
5620.72	Fe	0	00		2	

TABLE I—Continued

λ Rowland	Identification Rowland	Solar Intensity	Spot Intensity	Amount Increase Weak Arc	Amount Enhance- ment Spark	Remarks
5694.96		0	1	1		
5702.54		0	0-1	1		
5712.99		0	1	1		
5783.29		2	2-3	0-1		
5784.08		3	4	1		
5785.19		2	3	0-1		
5785.95		1	2-3	n.c.		
5788.14		4	5	1		
VANADIUM						
3910.98	<i>Fe-V</i>	4	5	1		
4090.73		1	2	1		
4090.94		2	3	1		
4105.32		2	3	1		
4115.33		3	4	1		
4116.63		1	2-3	1-2		
.71	<i>V, Fe?</i>	0				
4128.25	<i>Ce-V,-</i>	6d	7	1		
4232.76		00	1	0-1		
4330.19		0N	2	1-2		
4332.99		0	2	2		
4341.17		0	2	2		
4379.40		4	5	2		
4390.15		2	3	2		
4392.24	<i>V?</i>	1N	2	n.c.		
4395.41	<i>V, Zr</i>	2	3	1-2		
4400.74		1	2	1		
4406.81	<i>V-</i>	2	4	2		
4416.64		0	2	1		
4421.73		0	2	1-2		
4428.71	<i>V-Cr</i>	2d?	2	1		
4436.31		0	2	1-2		
4438.01		0	2	2		
4441.88	<i>V-</i>	3Nd?	5	2		
4444.57	<i>V-Ti</i>	00	2	1-2		
4452.17		0N	2	0-1		
4459.92		1	3	2		
4545.51	<i>Cr-V</i>	0	1	1-2		
4577.36		0	2	2		
4580.59		1	2	2		
4586.55		1	2-3	2		
4594.30		2N	4	2-3		
4635.35		00N	1	1		
4831.83		00	2	2		
4832.62		00	2	2		
4851.69	<i>Ca, V</i>	1	3	2-3		
4864.92		0	3	3		
4875.67		1	3	3		
4881.74		1N	3	3		

TABLE I—Continued

λ Rowland	Identification Rowland	Solar Intensity	Spot Intensity	Amount Increase Weak Arc	Amount Enhance- ment Spark	Remarks
5176.95		ooo	o	n.c.		
5625.10	—	ooo	0-1	1-2		H.
5626.25	—	oooN	1-2	n.c.		H.
5627.86		oo	3	2		
5632.67	—	oooNd?	o	0-1		H.
5646.32	—	oooN	1-2	1-2		H.
5657.67	—	ooo	1-2	1		H.
5668.59		ooo	2	1		
5671.07		o	3	1-2		
5698.75		1	4	2		
5703.80		1	3	2		
5707.20		o				
5707.26	Fe	1	3	1-2		
5727.27	Ti-V	2N	4-5	2		
5727.87	—	oo	3	1-2		H.
5731.44	—	oo	3-4	1-2		H.
5734.26	A?	ooo	1	n.c.		H.
5737.29	—	o	3	1-2		H.
5743.64	—	oo	3	2		H.
IRON						
4258.48		2	3-4			Widening probably due to 58.64 = Hasselberg's Ti 58.68
4508.46	Fe?, -	4	3		2	
4522.80	—	3	2	-1	2	L., Re.
4556.06	—	3	2		1	L., Re.
4584.02	Fe-	4	3	-1	3	
4603.13		6	7-8	1		
4630.31		4	5	n.c.		
4733.78		4	6	1		
4924.11		5	4	-1	3	
5018.63		4	3	-1	3	
5022.41		3	2	-1		
5110.57		5d	6-7	1		
5169.07		3				
5169.22		4	5-6	-2	4	
5198.80		3	4	0-1		
5218.37		1	o		2	Enhancement in spark may vary
5225.70		2	3-4			Widening probably due to 25.88 = Rowland's V 25.92
5250.82		3	4	0-1		
5260.72		8d?	10	2		
5276.17	Fe?	3				
5276.24	-Cr?	2	4	n.c.	0-1	
5316.79		4	2-3	-1	2	
5328.24		8d?	9	1		
5370.17		6	5	-1		
5371.66	Cr?	4				
5371.73		3	10	2		
5373.90	Fe, Cr	2	3	0-1		

TABLE I—Continued

λ Rowland	Identification Rowland	Solar Intensity	Spot Intensity	Amount Increase Weak Arc	Amount Enhance- ment Spark	Remarks
5381.22	<i>Fe?</i>	2	1			<i>Ti</i> line 5381.20 is enhanced 3
5397.34		7d?	8-9	1-2		
5404.36		5	4	-1		
5405.99		6	7	1-2		
5429.91		6d?	7-8	2	0-1	
5447.13		6d?	8-9	2	0-1	
5455.67		2				
5455.83		4	7	1		
5497.74		5	7	1		
5501.68		5	6	0-1		
5507.00		5	6-7	1		
5586.99		7	8	0-1		
5712.10		3	6			Hasselberg gives <i>Ti</i> at 5712.07
MANGANESE						
4762.57		5	6	1		
4823.70		5	6-7	1-2		
5255.49		od?	0-1	n.c.		
5377.80		2N	1	-1	0-1	
5394.84		1				
5394.91		1	4-5	2		
5407.59		0				
5407.69		0	2	1		
5420.51		0N				
5420.61		0N	3-4	1-2		
5432.75		1Nd?	3-4	2		
5470.80		0				
5470.88		0	3	1		
5506.10		1	1-2	n.c.		
5516.95		0				
5517.03		0	2	1		
5537.93		00				
5538.02		00	2	1		

abbreviation "H", under "Remarks," in the last column of our tables, indicates that Hasselberg gives a line at this point which we consider to be identical with the solar line in question. Similarly "L" and "Re" refer to the tables of enhanced lines published by Lockyer and Baxandall, and Reese, respectively. The spot intensities of the lines for the region extending from the violet to λ 5000 have been determined from photographs of the spectrum of the large spot of the latter part of June. The remainder are obtained from a combination of results from these plates with those from plates of numerous small spots taken previously.

TABLE II
 "ENHANCED" LINES IN ARC AND SPARK
 TITANIUM

λ Rowland	Identification Rowland	Solar Intensity	Amount Enhance- ment Spark	Amount Decrease Weak Arc	Remarks
3722.73	<i>Ti-Fe</i>	6	2	1	
3757.82	<i>Cr-Ti</i>	4	3	2	
3759.45		12d?	5	2	
3761.46		7	5	1-2	
3900.68	<i>Ti-Fe</i>	5	5	1	
3913.61	<i>Ti-</i>	5d?	5	1	
4012.54	<i>Ti, Ce</i>	4	2	0-1	
4028.50	<i>Ti-Ce</i>	4	3	1	
4053.98	<i>Cr-Fe-Ti</i>	3	3	n.c.	
4161.68	—	4	1-2	0-1	H.
4163.82	<i>Cr-Ti,-</i>	4	5	2	
4172.07	<i>Ti, Fe</i>	2	5	1	
4290.38		2	3	1	
4294.20		2	2	2	
4300.21		3	4	2	
4302.08		2	1-2	1	
4313.03		3	3	2	
4314.96		1	2	1	
15.14		3			
4316.96	<i>Ti?</i>	1	1	1	
4338.08		4	3	2	
4341.53	<i>Ti?</i>	2	1	1	
4344.45	<i>Ti-</i>	2	2	2	
4351.00		1	2	2	
4367.84		2	4	2	
4387.01	<i>Ti?</i>	1	3	1	
4395.20		3	4	1-2	
4399.94	<i>Ti, Cr</i>	3	2	2	
4411.24	<i>Cr-</i>	1	4	0-1	
4443.98		5	4	2-3	
4468.66	<i>Ti-</i>	5	4	2	
4488.40	—	1	3	1	H.
4501.44	<i>Ti, -</i>	5	4	2	
4534.14	<i>Ti-Co</i>	6	3	2	
4549.81	<i>Ti-Co</i>	6d?	5	2	
4563.94		4	4	2	
4572.16	<i>Ti-</i>	6	5	2	
4590.13	—	3	2		H.
4805.28	—	3	2	0-1	H.
4911.37	—	1	4		H.
5072.48		0	1		
5120.34	<i>Ti?</i>	3	3	1-2	
5154.24	<i>Ti-Co</i>	2	1	1	
5186.07		2	2	1	
5188.86		2	5	1	
5226.71	<i>Ti-</i>	2	4	2	
5336.97	<i>Ti,-</i>	4	3	2	
5381.22	<i>Fe</i>	2	1-2	1-2	H.
5418.98	<i>Ti?</i>	1	1-2	1	

TABLE II—Continued

CHROMIUM

λ Rowland	Identification Rowland	Solar Intensity	Amount Enhance- ment Spark	Amount Decrease Weak Arc	Remarks
3603.83	—	2	3	1	H.
3678.04	—	2	4	n.c.	H.
3715.32	—	2	3-4	1	
3865.67	Fe-C	7	3		H.
3979.66	Nd-Co	4	1-2	1	L.
4012.63	—	0	0-1	1	
4145.91	—	1N	3		L.
4179.41	Cr, Co	0	1	n.c.	
4225.02	—	2N	1-2		L.
4242.54	—	2	3		L.
4284.38	—	2Nd?	1	1	L.
4555.16	—	2	1	1	L.
4558.83	Cr?	3	4	1	
4588.38	—	3	3	1	H.
4592.23	Cr-	1	0-1		
4634.25	—	2	1	1	H.
4824.32	Fe	3	4	1	L.
4848.44	—	2	3		L.
4876.59	—	1	2		L.
5237.49	Cr?	1	3		
5335.05	Co	1	2		
5620.72	Fe	0	2		

VANADIUM

3899.28	—	2	4	2	H.
3903.40	V-Ce?	2	4	2	
3916.55	—	3	3	2	H.
3952.10	Mn, V	2	4	2	
3968.24	—		2	1	Hasselberg's λ
3973.80	Nd, V, Fe	1	3	2	
3997.26	Cr?	1	2	1	H.
4003.08	—	2	2	1	H.
4005.86	V	3	5	2	
4023.53	V, Co	3	4	2	
4035.77	—		4	2	Hasselberg's λ
4036.92	V,-	1	1	1	
4202.51	—	0Nd?	2	1	H.
4205.24	—	1	2-3	1	H.
4225.38	—	0	2	1	H.

IRON

4173.62	—	3	0-1	0-1	L.
4179.02	—	3	0-1	1	L.
4219.52	—	4	1	0-1	Re.
4233.33	Mn	4	1-2		L., Re.
4303.34	—	2	1		L., Re.

TABLE II—Continued

λ Rowland	Identification Rowland	Solar Intensity	Amount Enhance- ment Spark	Amount Decrease Weak Arc	Remarks
4385.55	—	2	1		L., Re.
4433.39	—	3	0-1	1	
4466.73	—	5	1	0-1	
4508.46	Fe?,—	4	2		
4515.51	—	3	2	1	L., Re.
4520.40	Fe?,—	3	1	0-1	
4522.80	—	3	2	1	L., Re.
4541.48	—	0	1		L., Re.
4549.64	—	2	3	1-2	
4556.06	—	3	1		L., Re.
4576.51	—	2	0-1		L., Re.
4584.02	Fe—	4	3	1	
4789.85	—	3	0-1	n.c.	
4924.11	—	5	3	1	
5018.63	—	4	3	1	
5169.07	—	3		2	
5169.22	—	4			
5276.17	Fe?	3	0-1	n.c.	
5316.79	—	4	2	1	
5429.91	—	6d?	0-1	-2	
5447.13	—	6d?	0-1	-2	

The second series of tables consists of a comparison of the intensities of the lines of these elements which are considerably enhanced in the spark, with their intensities in the weak arc. Most of these lines which occur in the less refrangible part of the spectrum are diminished in spots, and such lines will appear in both sets of tables. As is well known, however, the great majority of strongly enhanced spark lines occur in the violet and ultra-violet, where the spot lines seem to have the same intensity as the Fraunhofer lines. Accordingly, independent lists of these lines have been added, since the evidence afforded by them as to the relation of spark to weak arc is of extreme importance. Almost all of these lines are unidentified in Rowland's table—a result which is of course due to his use of the arc lines for purposes of identification. Consequently we have been obliged to identify many of them, and in the course of this work have used the tables of Hasselberg and the lists of enhanced lines by Lockyer and Baxandall, and by Reese, to great advantage. References to these lists are made in the same way as in the preceding tables.

We have attempted throughout these tables to include only those

cases of blends in which the element under which they are classified in each instance seems to be the predominant factor in producing the effect.

DISCUSSION OF THE TABLES

Before entering upon a discussion of the evidence afforded by these tables, it will be well to indicate to what extent they are to be regarded as comprehensive, and to examine the character of the lines not appearing in them. At the outset the important statement should be made that we have found very few lines which are certainly strengthened or weakened in the low-current arc that are not either spot lines or lines enhanced in the spark. There are, of course, in spot, in weak arc, and in spark many instances near the limits of accuracy of estimates of intensity, but for unquestionable cases the statement above seems to be true. To what extent the converse, that all spot lines are affected in the weak arc, the spark, or both, fails to hold, the tables indicate, and this side of the subject we shall discuss later in this paper.

The lines which are affected in spots and which do not appear in these lists fall into two classes. The first consists of lines so faint that they fail to appear upon our laboratory photographs, or are too weak to furnish quantitative results. The second class consists of lines for which the means of our determinations of change of intensity fall below what we consider the limits of accuracy. In the second class, accordingly, there should be no lines which are prominently affected in our photographs of spot spectra, and we believe this to be the case. The majority of the lines which are too weak upon our photographs to admit of satisfactory discussion belong to iron and chromium. Since any marked cases of strengthening in the low-current arc as compared with the strong arc would be visible for most of these lines, it is clear that such indirect evidence as they afford is in agreement with the fact that the fainter spot lines belonging to these elements are inconspicuous. The evidence furnished by lines enhanced in the spark, for which the absence of an entry in the column "Amount Decrease Weak Arc" indicates that the line is not visible in the weak arc, is considerably more important. Since most of these lines can be seen in the strong arc, it is clear that they are weakened in the low-current arc and that only quantitative data are lacking.

In order to simplify the analysis of these tables we shall discuss separately the cases of lines strengthened in spots, and those weakened in spots. An inspection of the results for lines of the first sort gives the following summary:

TABLE III

Element	No. of Lines Strengthened in Spots	No. of Lines Strengthened in Weak Arc	Unchanged Weak Arc
<i>Ti</i>	88	83	5
<i>Cr</i>	46	42	4
<i>Fe</i>	19	18	1
<i>V</i>	56	52	4
<i>Mn</i>	11	9	2
	220	204	16

The relatively small number of *Fe* lines in this summary is due to three causes: first, numerous *Fe* lines are weakened in spots; second, a large number of lines are only very slightly affected, if at all; third, many *Fe* lines are very faint in the arc spectrum, and do not appear on our plates.

The evidence afforded by this comparison of results certainly indicates very strongly the connection of spot lines with those strengthened in the weak arc. The exceptions form but seven per cent. of the total number, and it seems probable that many of these may be accounted for either by misidentification in Rowland's table or by the presence of close companions in the solar spectrum to which the change of intensity in the spot is due. In the table for iron three cases of the latter kind are indicated by notes, and no doubt others exist.

The relation of the lines weakened in spots to those in the spark and the low-current arc is one of the most interesting results of this investigation. The following brief table gives a summary for such lines:

TABLE IV

Element	No. Lines Weakened in Spot	No. These Enhanced in Spark	No. Not Changed in Spark	No. Diminished in Weak Arc	No. Not Seen in Weak Arc	No. Not Changed Weak Arc
<i>Ti</i>	8	8	0	8	0	0
<i>Cr</i>	10	10	0	6	4	0
<i>Fe</i>	13	10	3	9	3	1
<i>Mn</i>	1	1	0	1	0	0
	32	29	3	24	7	1

We have not included in the above list the interesting case of the line λ 5381.22. This line, identified by Rowland as *Fe*, is diminished in spots. No line is given here in the arc tables of Kayser and Runge, nor does it appear upon spark plates of this region. There is, however, a line of *Ti* given by Hasselberg at λ 5381.20 which is enhanced 3 in the spark, and this is probably identical with the solar line.

Further important evidence as to the relation between weak arc, strong arc, and spark is furnished by a number of lines which lie so far to the violet that they apparently are beyond the range of lines affected in sun-spots. This region is particularly rich in lines enhanced in the spark, and it has seemed desirable to consider here also the changes produced in passing from the strong arc to the low-current arc. The following summary derived from Table II includes the most important of these lines. For the sake of completeness the lines comprised in Table IV are also included.

TABLE V

Element	No. Lines Enhanced in Spark	No. These Diminished in Weak Arc	No. Not Seen in Weak Arc	No. Not Changed in Weak Arc	No. Strength- ened in Weak Arc
<i>Ti</i>	48	44	3	1	0
<i>Fe</i>	25	14	7	2	2
<i>Cr</i>	22	10	10	3	0
<i>V</i>	15	15	0	0	0
<i>Mn</i> ¹	1	1	0	0	0
	111	84	20	6	2

As stated before, we consider the evidence afforded by enhanced lines which fail to appear in the weak arc to be confirmatory of the conclusion that these lines are, in general, diminished in passing from strong to weak arc.

The two lines of iron which appear in the above table as increased in intensity both in spark and in weak arc are the strong yellow lines λ 5429.91 and 5447.13. Both lines are considerably increased in the weak arc, and both are strengthened in sun-spots. We are obliged at present to regard these lines as exceptions. Attention

¹ Manganese has no prominently enhanced lines in the region which we have investigated. The single line given occurs in the yellow, and it is but slightly affected.

should, however, be called to the fact that both are double in the Sun and probably in the arc as well. This may perhaps account for their contradictory behavior in the spark and the weak arc. The contradiction in the case of spots would, however, still exist, since visual observations in the third order of the 18-foot spectroscope have shown that the amount of strengthening of the two components for each of the lines is about the same, so far as can be judged with the resolving power available.

A consideration of the quantitative relations of the enhanced lines of Tables I and II shows that the general agreement between the amount of enhancement in spark and of decrease of intensity in low-current arc is reasonably good, in view of the obvious difficulties in the way of making a comparison on account of the marked difference in the magnitude of the effect in the two cases. The agreement between the amount of decrease in weak arc and in spot is still better, apparently, but too much emphasis should not be laid on this fact, since the range of effect is relatively small. The behavior of silicon, which we have not investigated as yet, should be especially interesting in this respect, its lines being weakened remarkably in sun-spots.

There remains one other feature of this investigation to which attention should be called. This has to do with the negative evidence and serves as a valuable check upon the results obtained. It is evident that in cases in which the intensities of lines in separate spectra are to be compared with each other the best criterion for the correctness of exposure time is given by the lines which are not affected in the two cases. In the present instance we have selected at random from the tables of arc lines for the various elements a series of prominent lines which are not affected in sun-spots. The behavior of these lines in the weak arc and in the flame has then been examined, and the results are indicated in Table VI.

The lines in the "doubtful" column all show some evidences of weakening, but the amounts are at the limit of accuracy of determination and of slight value.

This table affords two kinds of important testimony. The first bears upon the general validity of our hypothesis, which assumes the existence of lines which are affected neither in spots nor in the

TABLE VI

Element	No. of Lines	Strengthened in Weak Arc	Unchanged	Doubtful
<i>Ti</i>	37	0	32	5
<i>V</i>	39	0	38	1
<i>Fe</i>	30	0	26	4
<i>Cr</i>	27	0	20	7
<i>Mn</i>	19	0	17	2
	152	0	133	19

weak arc. The second goes to show that our quantitative values cannot be affected in any considerable degree by errors in the relative exposure times of our spectra.

In concluding this preliminary discussion of the observational materials it is well to call attention to one or two essential difficulties in the way of agreement between laboratory and solar results, arising from the nature of sun-spot spectrum photographs. The first of these is that in the sun-spot spectrum the blending of close lines or the superposition of lines of different elements is very liable to hide the true behavior of those under consideration. This is especially true in the blue and violet regions of the spectrum, where the number of lines in the solar spectrum becomes very great. The second difficulty arises from the fact that the number of lines shown to be affected upon a sun-spot photograph depends upon a variety of conditions, in particular the character of the definition at the time, and the size of the sun-spot. Accordingly, until a very large amount of observational material shall have furnished what will actually be a definitive list of sun-spot lines (assuming that the sun-spot spectrum itself does not vary essentially), it will be better to confine comparisons of sun-spot and laboratory results to the more important lines in each case.

TEMPERATURE AS THE PROBABLE CAUSE OF THE OBSERVED PHENOMENA

From the above discussion of the lines in our tables it appears that, in general, (1) the lines which are strengthened in spots are strengthened in the 2-ampere arc; (2) the lines weakened in spots are weakened in the 2-ampere arc; (3) the lines weakened in the 2-ampere arc are "enhanced" in the spark.

Although our work must be carried much farther before final conclusions can be drawn, it nevertheless seems probable that the observed differences of intensity, both in the arc and spark and in sun-spots, may be adequately accounted for by temperature differences. Accordingly, the simplest way of accounting for the characteristic phenomena of spot spectra seems to be on the hypothesis that the metallic vapors in the spot are cooler than the corresponding vapors in the reversing layer.

However, the production of enhanced lines has been ascribed to such diverse causes that we shall defer judgment as to the arc and spark changes until our experiments can be repeated with an electric furnace. The present discussion is consequently to be regarded as a preliminary one, serving to indicate why we are inclined to ascribe the observed phenomena to the effect of varying temperature.

1. In his paper on the synchronous arc Crew states unequivocally: "Increase of phase in these experiments undoubtedly means increase of temperature."¹ The gradual weakening and finally the complete disappearance of the carbon flutings as the phase was reduced; the decreasing intensity of H and K, and the simultaneous increase of the blue calcium line; the increasing relative intensity of the *Al* pair, as compared with the neighboring H and K lines; and the resemblance of the iron spectrum at zero phase to the flame spectrum; are typical of the phenomena observed by Crew. All of these criteria, and others as important, apply as well to our weak arc plates as to those obtained by Crew with the synchronous arc.² From this standpoint, therefore, there is no conflict of testimony: Crew's conclusion as to the comparatively low temperature of the low-phase synchronous arc should apply also to the 2-ampere direct-current arc, at least in so far as it depends on the above criteria.

2. In their investigation on the temperature of the arc, Waidner and Burgess found the temperature of the crater to be reduced 70° when the current was reduced from 30 to 15 amperes. They give no results for smaller currents. In view of the fact that the relative intensities of the lines undergo no material change in passing from

¹ *Astrophysical Journal*, 22, 201, 1905.

² Although the carbon flutings do not disappear in the 2-ampere arc, they are considerably weakened.

30 to 15 amperes, while the change between 30 and 2 amperes is very pronounced, it is probable that the temperature of the crater is considerably reduced at 2 amperes. As for the arc itself, the great reduction in brightness, demanding exposures of four minutes at 2 amperes as compared with 12 seconds at 30 amperes, is strong evidence in favor of a lower temperature, since we know that the total radiation varies as the fourth or fifth power of the temperature. It should be recognized, however, that the change in brightness may be due in considerable part to the change in the amount of luminous vapor.

3. Since the enhanced lines of the spark appear with lower intensity in the 30-ampere arc, and are still further reduced in passing to the 2-ampere arc, no explanation hitherto advanced to account for these lines appears adequate in the present case, unless it be the explanation based on change of temperature. However, the fact that for the observed lines the changes of intensity with current strength are the reverse of those described by Hartmann in certain other cases, indicates the necessity of caution. For example, Hartmann found the enhanced line $Mg \lambda 4481$ to be greatly strengthened in the arc, as the current was reduced from 8 to 0.4 amperes. He concluded that "the condition for the development of those molecular vibrations to which the line $\lambda 4481$ corresponds were much more favorable in the small arc in spite of its lower, or at least certainly not higher, temperature than in the larger arc."¹ In order to keep the 0.4-ampere arc burning, it was necessary to start it several hundred times during the exposure. Hartmann considers it probable that $\lambda 4481$ was produced only at the moment of make or break. Sir William and Lady Huggins also ascribe the production of this line to a sudden discharge,² and Crew concludes that in a constant voltage circuit small currents, which heat the electrodes less, will result in a higher voltage between the electrodes, and probably a much quicker break than larger currents. According to him, the essential condition for the appearance of spark lines in arc spectra is a high and rapidly changing E. M. F.³ It may be mentioned here that our exposures to the weak arc were never commenced until it

¹ *Astrophysical Journal*, 17, 273, 1903.

² *Ibid.*, 17, 145, 1903.

³ *Ibid.*, 20, 281, 1904.

was burning steadily, after the poles had been separated sufficiently to prevent all light except that from the arc proper from entering the short slit.

4. The behavior in stars of the lines affected in sun-spots appears to be consistent with the view that temperature changes alone are sufficient to account for their variation in intensity.¹ In *α Orionis*, which we have other reasons (such as the great strength of the blue calcium line and the presence of titanium oxide flutings) to regard as much cooler than the Sun, lines that are strengthened in sun-spots are still further increased in intensity.² In *Arcturus*, which is always assumed to have a temperature between that of *α Orionis* and the Sun, the intensities of the lines agree remarkably with those observed in sun-spots.³ We hope in a future paper to discuss this subject more fully, especially in its bearing on stellar evolution and the classification of stellar spectra.⁴

5. A bolographic study of spot spectra, on which Mr. Palmer is engaged, indicates that the maximum of intensity is very considerably shifted toward the red, as compared with its position in the spectrum of the photosphere. This result is thus in harmony with the view that the temperature of the vapors in spots is lower than that of the photosphere.

6. The application of our results to chromospheric lines may prove to be of importance, but we are not yet in a position to enter into a full discussion of the subject. In a recent paper⁴ Fowler has called attention to the weakening of a number of enhanced lines of iron, titanium, and chromium in spots, these lines having been taken from a list of "long or high-level" chromospheric lines. Our observations of the large spot of June, which was particularly favorable for determinations of this sort, give results in close agree-

¹ Formerly we were inclined to the view that the presence of spot lines in the spectra of red stars indicated the presence of spots like those on the Sun. Our recent work has led us to the opinion that the comparatively low temperature of these stars offers the simplest explanation of the observations.

² Hale and Adams, *Contributions from the Solar Observatory*, No. 8, *Astrophysical Journal*, **23**, 400-405, 1906.

³ Adams, *Contributions from the Solar Observatory*, No. 12, *Astrophysical Journal*, **24**, 69-77, 1906.

⁴ *Monthly Notices*, **65**, 361, 1906.

ment with his for these lines. From the evidence discussed in the present communication, however, we should consider the behavior of these lines to be explained much more simply and satisfactorily on a temperature basis than on any which involves the conclusion that they originate at a higher level in the chromosphere than most other lines of the spectrum. In regard to this question, indeed, we see no occasion to differ from Evershed's conclusion¹ that "there is no evidence of differences in the relative intensities of the lines of an element in the higher or lower regions of the flash layer, and the enhanced lines appear to predominate throughout the entire depth of the radiating stratum."²

SUMMARY

1. This paper describes a preliminary study of the more important sun-spot lines in the region above λ 5800, belonging to titanium, chromium, iron, vanadium, and manganese—the metals most characteristic of sun-spots.

2. Over ninety per cent. of the lines in our tables, which are strengthened in sun-spots, are found to be strengthened in passing from a 30-ampere arc to a 2-ampere arc.

3. Over ninety per cent. of the lines shown by our tables to be weakened in sun-spots are weakened or absent in the 2-ampere arc.

4. Over ninety per cent. of all the "enhanced" lines included in our tables are weakened or absent in the 2-ampere arc.

5. In a list selected at random of 152 lines which are not spot lines no cases were found of lines strengthened in the low-current arc or in the flame.

6. We are not yet ready to express a final opinion, but are inclined to the view that temperature differences are adequate to account for the above phenomena. Our reasons for this view may be summarized as follows:

¹ "Solar Eclipse of 1900, May 28," *Philosophical Transactions*, Series A, 201, 477.

² Bright lines, due to overhanging flocculi or to eruptive phenomena, frequently appear in spot spectra. Indeed, H_3 and K_3 are always present. The weakening of the enhanced lines in spots, however, should in our opinion be classed with the strengthening of other lines, and attributed to the same cause.

- a) The similarity of the spectroscopic phenomena of the weak arc to those of the low-phase synchronous arc, held by Crew to correspond to a low temperature.
- b) The probable decrease in the temperature of the arc with decreasing current strength.
- c) The behavior of the enhanced lines in the 2-ampere arc.
- d) The presence of sun-spot lines in red stars.

ADDENDUM

When the above paper was written, it was not supposed that the competence of temperature differences alone to account for the observed phenomena could be tested at present. It was subsequently decided, however, to construct an electric furnace for immediate use in our Pasadena laboratory, where sufficient current is available. A Littrow spectrograph was also built for observations with the furnace. This is supplied with a combined collimating and camera objective, by Zeiss, of 6 inches (15 cm) aperture and 18 feet (5.49 m) focal length, and a Rowland plane grating, with ruled surface 5×3.75 inches (12.7×9.5 cm), having 15,000 lines to the inch (5,900 to the cm), for the use of which we are indebted to the kindness of Professor Ames.

In addition to our work with the furnace, which leaves no doubt that the arc phenomena can be accounted for as effects of temperature, we have obtained independent evidence, by observations of the outer portions of the flame of an ordinary arc, which leads to the same conclusion. In this investigation the metal under consideration, in the form of a powder, was placed in the positive pole. The vapor in the long flame which rises from such an arc is undoubtedly of lower temperature than that between the poles. Photographs of the spectrum of the flame show the bands of the oxide, and give other evidences of low temperature, such as the strengthening of the blue calcium line, which is more intense than H and K. The spectrum of the flame, compared with that of the core of the arc (between the poles), shows changes of line intensity similar to those observed with the 2-ampere arc and the synchronous arc.

In order to test fully the important question of the identity of these variations, we have made an extensive series of comparisons

of the weak arc and flame. An independent set of estimates for the lines affected in the flame was first made, and this was then compared with the determinations for the lines affected in the weak arc. The results of this comparison are indicated by the following summary:

TABLE VII

Element	No. Lines Compared	Affected in Same Direction	Affected Same Amount	Amount Greatest Difference
<i>Ti</i>	122	111	98	0.5
<i>V</i>	49	42	34	0.5
<i>Cr</i>	62	57	51	1.0
<i>Fe</i>	28	24	20	0.5
<i>Mn</i>	12	12	10	0.5

The quantities in the last column are in terms of units of the scale, extending from 0 to 5, used in the estimates of differences of intensity in our previous tables.

It is evident from this comparison that there can be little doubt as to the identity of the effects found to exist in the low-current arc and in the outer portions of the flame of the strong arc. In consequence we may say, in general, that a large majority of the lines that are strengthened in sun-spots are relatively strengthened in the flame, while a similar proportion of those that are weakened in sun-spots are relatively weakened in the flame.

The second part of the evidence is furnished by some results obtained with the electric furnace. This work was undertaken in the hope of observing such variations of intensity as were found in the weak arc, under conditions which would eliminate all possible electrical effects and leave temperature as the only possible agent for producing them.

The furnace used was of a type employed by Moissan. A carbon tube of $1\frac{1}{4}$ inches (3.2 cm) external and $\frac{3}{4}$ inch (1.9 cm) internal diameter was passed through the walls of a small rectangular box of retort carbon, imbedded in a large mass of powdered magnesite. At right angles to this tube were placed the carbon poles of a powerful arc, the center of the opening between the poles coming just above the carbon tube. Accordingly, when the current was passed between the poles, the arc played on and over the tube, and the intensely

heated chamber assisted in distributing the heat over a considerable portion of its length. A current of about 600 amperes at a potential of 50 volts has been used, and with few modifications we expect to be able to employ a similar amount of current at 100 volts. The element to be investigated is placed in the interior of the carbon tube and the light from the opening is projected by a lens upon the slit of the Littrow spectrograph.

With this instrument we have photographed the spectra of iron and manganese, and expect soon to obtain that of chromium as well. It is not yet certain whether it will be possible with this form of apparatus to vaporize the highly refractory elements titanium and vanadium, but we hope to be able to do so with the aid of the higher potential.

The comparison of the plates of manganese and iron in the green and yellow regions with those obtained in the flame and in the low-current arc has proved of the greatest interest and importance. An independent estimation of the relative intensities of the manganese lines upon the furnace plates showed that all ten of the lines of wavelength greater than 5255 given in Table I are affected. Of these seven are affected in the same direction and by the same amount. The enhanced line λ 5377.8 is reduced 2 in the furnace as against 1 in the weak arc; and the lines most strongly affected in the weak arc, λ 5395 and λ 5433, are so greatly strengthened in the furnace as to be reversed. The results given by the iron plate are similar. Of the twenty-four lines to the red of λ 5110 three are too faint upon the furnace plate to be visible. The remaining twenty-one lines are affected similarly in furnace and weak arc, and seventeen of these by the same amount. The largest difference among the other four amounted to but one-half of one division.

OBJECTIONS TO THE TEMPERATURE HYPOTHESIS

A summary of the principal objections to the hypothesis that difference of temperature is the essential factor in producing the phenomena observed by us in flame, spark, and sun-spots will form a desirable corollary to the summary which concluded our main discussion. It will also be of material interest as indicating the directions along which we expect to continue most of our investigations.

The more important of these objections are as follows:

1. The comparative absence in sun-spots of strengthened and weakened lines in the blue and violet portions of the spectrum. While it is true that the differences in the relative intensities of lines in the weak arc and in the strong arc become as a rule less in the more refrangible parts of the spectrum, there are still some marked cases of strengthening in the weak arc of lines which are not affected in sun-spots. Perhaps the most important instances of this sort are the three *Cr* lines λ 4290, λ 4275, and λ 4255. All of these lines are decidedly strengthened in the weak arc and the flame, but are not affected in spots.

To the same category of lines belongs the blue line of *Ca* at λ 4227. This increases in intensity with a diminution of temperature. It does not, however, seem to be affected in spots.

In the case of enhanced lines the matter is still more striking. As is well known, the strongest enhanced lines of nearly all of the elements are found in the ultra-violet. As against this is the fact that the most refrangible line which we have yet observed upon our photographs as unquestionably affected in sun-spots is λ 3906.

Though we have at present no sufficient explanation for this condition of affairs, there are one or two considerations which seem to bear upon it. One of these is the striking similarity in this respect of a photograph of an abnormal solar spectrum obtained in 1894 at the Kenwood Observatory.¹ On this plate the character of the solar spectrum from λ 4100 to λ 3900 is totally changed. Beyond λ 3900, however, the spectrum gradually returns to the normal solar type. While the analogous behavior of sun-spot spectra may be no more than a coincidence, the similarity calls for remark. Furthermore, in view of the fact that certain elements, such as vanadium, manganese, and, in general, titanium, show very close agreement with our hypothesis, while others show important exceptions, the possibility is suggested that the various elements are differently distributed in spots, and that the vapors are not all of the same temperature.

A tentative explanation of the above difficulty might be founded on the hypothesis that the vapors producing the greater part of the characteristic changes in sun-spot lines occur at the base of the

¹ *Astrophysical Journal*, 16, 211-233, 1902.

reversing layer, where they may form a comparatively thin stratum. Consequently, in the more refrangible part of the spectrum, the absorption of the overlying vapors may be so great as to mask almost completely the selective absorption of this low layer. From this point of view the spectrum of a spot in the more refrangible region would be due mainly to the higher vapors, and would show few differences from the ordinary solar spectrum. The very great falling off in the intensity of the spectrum of the spot as compared with disk in the violet region, where the intensity is only about one-third to one-fifth as great as in the yellow and green regions, probably indicates the action of powerful general absorption. On the above hypothesis this absorption is assumed to be sufficient practically to obliterate the effects of selective absorption, the vapors producing the latter being assumed to be restricted in depth.

2. Some lines in the less refrangible region of the spectrum seem to be strengthened in the spark at the same time that they are strengthened in the flame and in spots. The principal cases of this sort are the iron lines λ 5430 and λ 5447.

In the same region of the spectrum occurs the titanium oxide fluting, the presence of which in third-type stars is recognized as an evidence of comparatively low temperature. This fluting is found in the spectrum of the strong arc as well as in the flame and the weak arc. Its apparent absence in the spectrum of sun-spots probably indicates a higher temperature for them than for the laboratory sources.¹

3. The changes in the relative intensities of lines in the flame do not in all cases harmonize in amount with those observed in spots. Examples of this sort are evident from an inspection of the tables. Examination of these tables also furnishes some indication of a greater amount of strengthening in the low-current arc of the strong lines than of the weak lines. The evidence is by no means to be considered conclusive, since the exceptions are numerous and important. That the effect, however, is not due to photographic or physiological causes is shown by its persistence upon plates taken with widely

¹ Since the above was written we have found the titanium oxide fluting, which begins at λ 5597.9, in one of our photographs of a sun-spot spectrum. Two maxima of the fluting are clearly visible. It is now probable that other titanium oxide flutings will be found on our plates.

varying exposure times. If genuine, it is identical with that claimed by Fowler to exist in the case of spot lines, namely, that such lines are in general affected in spots in proportion to their absolute intensities.¹ This conclusion we had occasion in a former communication to oppose,² and it certainly is not borne out by the results for at least two of the elements most prominent in sun-spots, titanium and vanadium. The question of the existence of such an effect for lines strengthened in the low-current arc and in the flame we shall hope to settle definitely at an early period in our future investigations.

In this connection, however, it is well to call attention to the fact that in the laboratory work and in sun-spots we are dealing with what are probably considerably different absolute temperatures. Consequently the different increment of change at different absolute temperatures may well account for some of the discrepancies mentioned above. If it were possible to determine by laboratory experiments the temperature of the spot vapors, further work along this line would be much simplified. Assuming the correctness of our hypothesis, such a method of determining this temperature, though perhaps subject to serious practical difficulties, would be by establishing the equality in the laboratory of a pair of sun-spot lines, one of them an enhanced line, which have been observed as of equal intensity. Since enhanced lines diminish in intensity as the temperature falls, while the other lines increase, equality of intensity in the laboratory should serve to determine the temperature in question.

4. As it seems to be the opinion of several observers that enhanced lines may be produced, at low temperatures, by suitable electrical means, this phase of the subject will require further investigation. Our hypothesis does not maintain, however, that temperature differences offer the *only* means of explaining the observed changes in metallic spectra. It merely assumes that variations of temperature afford a simple and satisfactory way of accounting for the changes in the relative intensities of the spot lines, as observed in the laboratory, in sun-spots, and in stars.

5. The hydrogen lines, as far as $H\delta$, are weakened in our photographs of spot spectra. This fact, taken in conjunction with the

¹ *Monthly Notices*, 65, 205-218, 1905.

² *Astrophysical Journal*, 23, 11-44, 1906.

great strength of the hydrogen lines in the chromosphere and in the first-type stars (where the enhanced lines are strengthened), and their weakness in red stars, might be made the basis of an hypothesis to account for the weakening of the enhanced lines in red stars and in spots. For Hartmann has shown that the enhanced lines are strengthened in an arc in water, because, in his opinion, of the hydrogen produced by its decomposition. A similar effect of hydrogen atmosphere has been observed by Crew with a rotating arc, but he ascribes the production of the spark lines to a more rapid break, caused by the presence of the hydrogen, which introduces an extra electromotive force.¹ We have observed the spectrum of a 110-volt continuous-current arc in hydrogen between fixed metallic poles. As there was no break, Crew's explanation does not seem adequate to account for the apparent strengthening of the enhanced lines on our photographs. If the results of this preliminary experiment are confirmed by later work, the hydrogen hypothesis may require further investigation.

It may be recalled that in his paper on "The Evolution of Solar Stars"² Schuster suggests the possibility that the extensive hydrogen atmospheres of first-type stars might account for their high temperatures, if it could be shown that hydrogen, perhaps in some form with which we are unfamiliar in the laboratory, absorbs strongly in the infra-red. We are unaware of any evidence pointing to the existence of such absorption, and therefore do not feel at all certain that the high temperatures of first-type stars depend upon their extensive hydrogen atmospheres. Whether the presence of hydrogen can produce an increase in the temperature of arc or spark may perhaps be determined by laboratory experiments.

AUGUST, 1906

¹ *Astrophysical Journal*, 20, 280, 1904.

² *Ibid.*, 17, 165-200, 1903.

A 100-INCH MIRROR FOR THE SOLAR OBSERVATORY

By GEORGE E. HALE

I am permitted to announce that Mr. John D. Hooker, of Los Angeles, has presented to the Carnegie Institution of Washington the sum of forty-five thousand dollars, to be used to purchase for the Solar Observatory a glass disk 100 inches (2.54 m) in diameter and 13 inches (33 cm) thick, and to meet other expenses incident to the construction of a 100-inch mirror for a reflecting telescope of 50 feet (15.24 m) focal length. These expenses will include the erection of a building in which the mirror can be ground, figured, and tested; the construction of a large grinding-machine, with crane for lifting the mirror; the provision of a 54-inch (1.37 m) glass disk, to be made into a plane mirror for testing purposes; the purchase of glass disks for the various plane and convex mirrors required in the telescope, etc. The optical work will be done by Professor G. W. Ritchey and the assistant opticians employed under his direction by the Solar Observatory.

In making this gift, Mr. Hooker's desire is to secure the realization of the great possibilities in astrophysical research which a large reflector seems to offer. He has absolute confidence in the ability of Professor Ritchey to make an essentially perfect mirror 100 inches in diameter; no one could ask better evidence of this than his gift affords. He knows, also, that in several classes of work, such as the measurement of the heat radiation of the stars, and the spectroscopic study of the faintest objects, the mirror is sure to yield results fully commensurate with its great size. But he is nevertheless aware that for certain other classes of work, in which the most perfect definition is essential to the highest success, the construction of a mirror of such great aperture must be regarded as an experiment. The immense block of glass will weigh four and one-half tons—four and a half times as much as the disk of our 60-inch (1.52 m) mirror. The difficulty of providing a mounting capable of carrying it with the necessary precision is not slight. The glass is certain

to be distorted by temperature changes, which would ruin its performance if not obviated. The atmospheric conditions, even on Mount Wilson, may not be sufficiently good to permit so great an aperture to be used to full advantage. Of these and other obstacles Mr. Hooker is fully informed, and he does not underestimate their importance. But he perceives and appreciates, with the understanding of one who has himself invented and developed mechanical appliances, that experiment is necessary to progress. He therefore does not hesitate to provide the means for undertaking an optical experiment on a large scale. Let us consider its probable outcome.

In the first place, the question arises whether a sufficiently homogeneous glass disk of the required dimensions can be obtained. Our long experience with the Plate Glass Company of St. Gobain leads us to believe that no insuperable difficulty will be encountered. This old and reliable company has cast for us scores of disks, from which Professor Ritchey has made a large number of plane and concave mirrors, from the smallest sizes up to 60 inches. At present the 60-inch is receiving the finishing touches in our optical shop, and two 36-inch (0.91 m) mirrors, for testing purposes, are well advanced, in addition to several large plane and convex mirrors for the 60-inch reflector. In all of these cases the glass disks furnished by the St. Gobain Company have left nothing to be desired. The 60-inch disk, 8 inches (20.3 cm) thick, and weighing a ton, is fully equal in quality to the smaller ones. We are therefore inclined to believe, since the St. Gobain Company expresses its deliberate opinion that a satisfactory disk, 100 inches in diameter and 13 inches thick, can be produced, that they will be able to carry out the order we have given them.

As for the work of grinding and figuring, no one who has watched the progress of our 60-inch mirror would be likely to doubt Professor Ritchey's ability to accomplish this difficult task. The method of parabolizing which he has recently perfected will apply as well to a 100-inch mirror as to the 60-inch. It eliminates the necessity of any hand-work, and has already yielded a paraboloidal figure so perfect that almost any other optician would be more than contented with it. Professor Ritchey rightly believes, however, that a still higher degree of perfection will be worth attaining, since its advan-

tages will be felt under the most perfect atmospheric conditions. I am confident that he will find no difficulty in bringing the 100-inch mirror, as well as the 60-inch, to this highest order of perfection.

The mounting should offer no great obstacles, especially as it will not be designed until the mounting of the 60-inch has been thoroughly tested on Mount Wilson. Unless I am greatly mistaken, this latter instrument will meet our best expectations. Professor Ritchey has taken the greatest pains with the design, and the co-operation of the able staff of engineers at the Union Iron Works has been most useful. The mechanical execution of the parts is admirable, and with the heavy machinery available, a mounting much larger than that required for the 100-inch mirror should be easily within the bounds of possibility. To a firm which has built some of the most powerful battleships and cruisers in our navy such a mounting would appear much less formidable than to the average instrument-maker, accustomed to a different class of work. Fortunately, the ideas of the Union Iron Works Company, as to the degree of precision required, are entirely in harmony with our own, and appear to have been met in the mounting of the 60-inch, which has just arrived in Pasadena.

The prevention of change of figure due to changing temperature should not prove a very serious problem. During the fine nights of the best observing season on Mount Wilson the temperature remains almost perfectly constant after 9 P. M. It will therefore only be necessary to maintain the mirror (or possibly the entire telescope) at approximately this temperature throughout the day, by means of suitable refrigerating machinery. In the long periods of absolutely cloudless weather the change of temperature from night to night is extremely small, so that little difficulty should be encountered on this score. If the slowly falling temperature during the early evening should prove to give trouble, the observational work might be deferred until after nine o'clock. The dome and building like those designed for the 60-inch reflector, will be so constructed that no air can enter during the day; they will also be protected by louvers from the heat of the Sun. The problem is, of course, altogether different from that encountered in the case of the Snow telescope, where the mirrors are required to give good images in spite of their exposure to direct sunlight.

Assuming that these various difficulties can be successfully overcome, it still remains a question whether the atmospheric conditions on Mount Wilson will be sufficiently good to permit the telescope to give satisfactory images. This cannot be definitely determined until after the 60-inch reflector has been used for some time. Even if it should prove, however, that only a very few nights in the course of a year can be utilized to the fullest advantage, the construction of such a telescope would nevertheless be desirable. For under the ordinary conditions, which are much finer than those in the eastern part of the United States, results of the highest value can be obtained in many classes of work, such as the photography of stellar spectra, the measurement of the heat radiation of the stars, etc. The immense amount of light which this mirror will collect should render it particularly suitable for spectroscopic work of all kinds.

It need hardly be said that the 100-inch mirror, when suitably mounted, will play a most important part in the scheme of research of the Solar Observatory. The investigation of stellar evolution, upon which we are engaged, frequently calls for adequate spectroscopic study of stars beyond the reach of existing instruments. In my work on the red stars of Secchi's fourth type, with the 40-inch Yerkes telescope, I encountered this difficulty, in spite of the great light-collecting power of that instrument. It was impossible to obtain satisfactory evidence as to the transition from solar stars to those of the fourth type. The large number of stars within the reach of a 100-inch reflector should greatly increase the possibility of finding the intermediate types, which are so important in their bearing upon the relationship of solar and red stars. This is only a single instance, but it forcibly suggests itself when considering our program of research. In other fields the large reflector should be equally valuable, especially for the photography of the numerous small spiral nebulae, the details of which should be brought out to good advantage with a focal length of 50 feet; minute study of the large nebulae, in the hope of detecting changes in their form; the study, with very high dispersion, of the spectra of bright stars, etc. The remarkable calm of the summer nights on Mount Wilson should assist materially in all of this work, since vibration of the tube, caused by the wind, would undoubtedly be a serious drawback under less favorable conditions.

No provision has yet been made for the mounting and dome. It is not known from what source funds for this purpose will come, but I believe a donor will be found by the time they are needed. Mr. Hooker's gift is very opportune, because of the fact that it permits us, now that the 60-inch mirror is nearly completed, to retain and use to the best advantage the services of the opticians trained by Professor Ritchey for our present work. The making of the glass disk, and the grinding and figuring of the various mirrors, will probably occupy about four years. Since the Union Iron Works Company will require only a year for the construction of the mounting and dome, it is evident that no funds for this purpose will be needed at present, and that the experience gained with the 60-inch reflector can be utilized in designing them.

PURCHASE OF THE SNOW TELESCOPE BY THE SOLAR
OBSERVATORY

As stated in previous papers, the authorities of the University of Chicago were kind enough to loan the Snow telescope to the Solar Observatory for a period of two years. It subsequently became the opinion of all parties concerned that, in view of the very satisfactory performance of this instrument on Mount Wilson, it would be advisable to keep it there permanently. Accordingly, the Snow telescope has been purchased by the Solar Observatory, and will thus form a part of its permanent equipment. I wish to express my sense of obligation to Miss Snow, to Professor Frost, Director of the Yerkes Observatory, and to Acting President Judson and the Trustees of the University of Chicago, for the courtesies shown us in connection with the loan and sale of this valuable instrument.

SEPTEMBER 1906.

MINOR CONTRIBUTIONS AND NOTES

THE SNOW FUND OF THE YERKES OBSERVATORY

In view of the very satisfactory performance of the Snow horizontal telescope during the year that it had been in operation at the Solar Observatory on Mount Wilson, the advisability of its permanent retention there suggested itself, some months ago, independently to the directors of the two institutions concerned. A less satisfactory performance had been obtained while the instrument was provisionally installed in its wooden house at the Yerkes Observatory; and it was not to be expected that any great improvement in that respect could be secured without a considerable expenditure in remodeling the building, if indeed the full capacity of the instrument could ever be realized in this climate and under the atmospheric conditions prevailing here.

The donor of the telescope and building, Miss Helen E. Snow, of Chicago, promptly acceded to my suggestion that, under the circumstances, the interests of all parties concerned might be furthered by the transfer of the Snow telescope to the Solar Observatory of the Carnegie Institution, and that the monetary compensation for the telescope be used as a fund for the purchase of auxiliary instruments for the Yerkes Observatory. The Acting President and Trustees of the University of Chicago also took action in accordance with this recommendation, and the Snow Fund has been established for this Observatory. After the purchase of a stereo-comparator, the principal of the fund, then amounting to about five thousand dollars, will remain intact, and the income will be added to the regular appropriations for the purchase of equipment at the Observatory. The Snow Fund constitutes the first item of endowment in connection with the Observatory.

The small coelostat used by our party at the eclipse of 1900 will be set up in the Snow building for solar work.

EDWIN B. FROST

YERKES OBSERVATORY,
September 1906

NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed.

Authors are particularly requested to employ uniformly the metric units of length and mass; the English equivalents may be added if desired.

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